

Saimaa University of Applied Sciences  
Technology, Lappeenranta  
Double Degree Programme in Construction and Structural Engineering  
Bachelor

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# **REALIZING THE BENDING STIFFNESS OF STEEL BEAM WITH ADDITIONAL DOWNSTAND PROFILES**

Thesis 2019

## **Abstract**

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Realizing the bending stiffness of steel beam with additional downstand profiles,  
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The objective of the research was to determine the influence of additional steel elements to the bending stiffness of a steel member at the erection stage. Also the aim of the study was to consider different designs of downstands welding on a steel member.

The data for this thesis were collected from Finnish norm and regulations, technical manual for Deltabeam® design taken from the Internet and by interviewing structure engineers at Peikko Finland Oy.

The results of the study show that the influence of downstands to bending stiffness of steel member is neglectfully small. Consequently, there is no necessity to include them in the calculations which help to simplify the estimation.

According to the results a future study of downstands regarding to bending stiffness of steel cross-section is not required.

Keywords: bending stiffness, influence of downstands, deflection

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## APPENDICES

Appendix 1	Deltabeam® dimensions
Appendix 2	Proof of width equivalence

# 1 INTRODUCTION

Currently, engineers use plenty of different types of structures and design solutions in the construction. It is possible because of long experience and numerous studies in this field.

Construction and structural engineering are developing all over the world by the virtue of its diversity. There are companies that developed their own unique design solutions and products.

This thesis was written for Peikko Finland Oy, which is an international company of building products. The company produced various kind of structural elements from joints and connections to reinforcement systems and floor structures.

One of the products is a composite beam, Deltabeam®, the cross-section which of is complex. This research is aimed to study steel member behavior of Deltabeam® more closely. It should help to review the accuracy of calculations relative to bending stiffness.

## 2 DELTABEAM® DESCRIPTION

Deltabeam® is a composite slim-floor system that consists of two elements: steel member and concrete filling. The behaviour of Deltabeam® depends on the building phase that is described below:

- Erection stage:  
Deltabeam® acts like a steel beam before the infill concrete has hardened
- Before/After load applying:  
Deltabeam® acts like a composite structure.

For this research a steel member of Deltabeam® was considered and calculated as a hollow steel beam. The components of Deltabeam® are shown in 1 Figure.

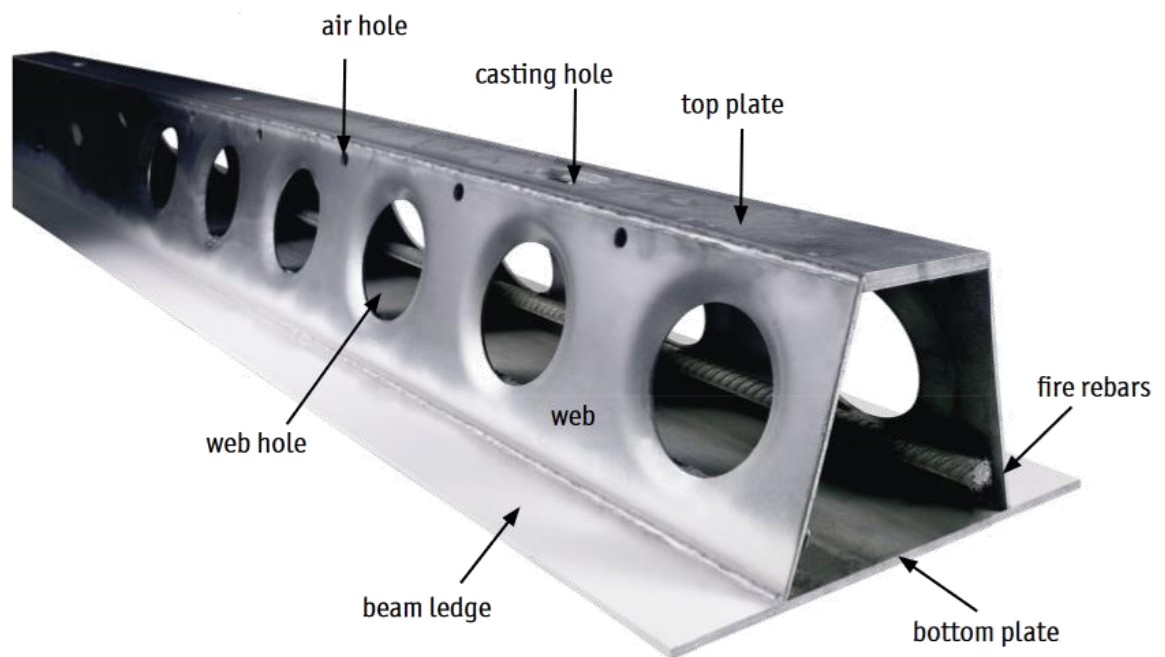


Figure 1. Components of Deltabeam® (1)

## 2.1 Deltabeam® types

There are two types of Deltabeam®: D-type and DR-type.

The distinctive features of these types are stated below:

- D-type:
  - beam has ledges on both sides which allows to carry loads on two sides
  - possibility of usage as edge beam
- DR-type:
  - beam has a vertical ledge on one side
  - beam carries loads on one side, for that reason used as edge beam.

Figure 2 presents Deltabeam's® appearance.

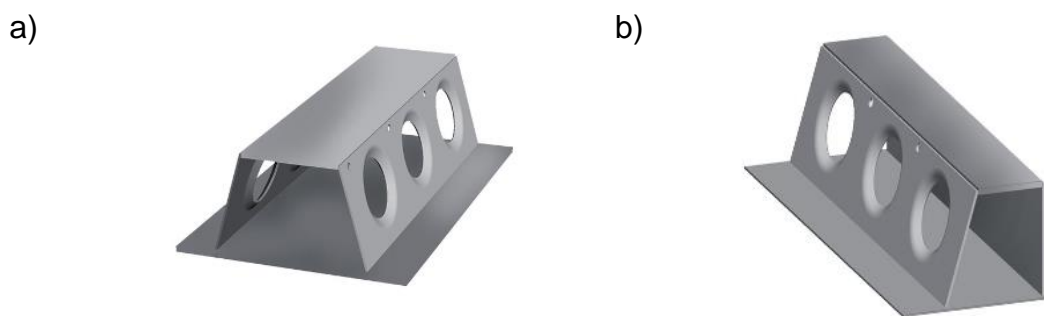


Figure 2. Appearance of Deltabeam®. a – D-type, b – DR-type (1)

Deltabeam® dimensions are indicated in APPENDIX 1.

## 2.2 Downstands

Downstand is a cold formed L-profile element used to carry loads from the slab to the ledge of the beam. These members are usually installed under different heights of slab and carrying beam.

Downstands are welded to the ledges of the beam (Figure 3).

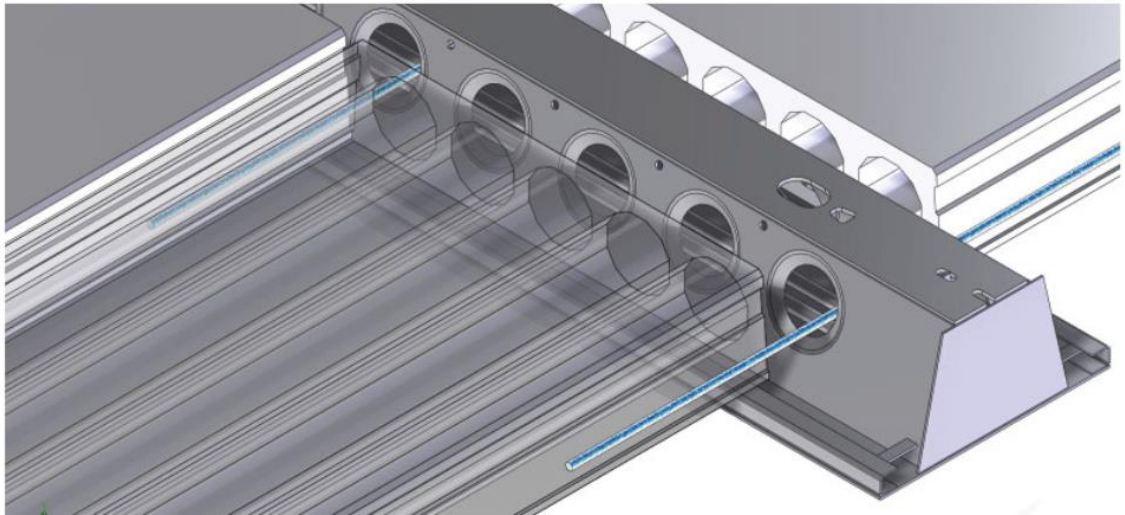


Figure 3. Deltabeam® with downstands on beam ledges (1)

## 2.3 Structural behavior

There are two different structures of Deltabeam® and it depends on the building phase as described above. Consequently, there are two ways how the beam acts and transfers the applied loads:

1. Erection stage: beam acts as hollow steel beam. Loads are transferred directly through beam ledges to Deltabeam®
2. Final conditions: the structure is a composite beam.

The steel member is analyzed in this research. For this reason only the influence of downstands and steel member elements to bending stiffness are considered to estimate the contribution to deflection.

Other than those listed parameters and possible reasons are mentioned below:

1. welding heat of downstands
2. welding method (fillet welds all around/ intermittent fillet welds)
3. welding materials (filler metal).

The bending stiffness is the function of elastic modulus  $E$  and the moment of inertia  $I$  of cross-section. The elastic modulus depends on the steel strength and other physics and mechanical properties of the material. The values of steel properties are listed in standards. The moment of inertia is a geometrical property of the section and it should be calculated according to Steiner's rules.

The steps of calculation due to Steiner's rule:

1. Define member cross-section axis relative to the calculations
2. Define axis relative to the center of gravity for each element of the section
3. Calculate the distance between mass center of element and chosen member axis
4. Calculate the distance between the chosen axis and the neutral axis
5. Calculate the moments of inertia for each element and sum them.

Calculations with detailed explanations should be considered as a separate topic.



### 3 CALCULATION OF MOMENT OF INERTIA

Downstands affect the mechanical strength characteristics of the beam. The aim of this research is to estimate the investment of L-profiles in bending stiffness of Deltabeam® steel member.

Assumptions for calculations are listed below:

1. Fastening of web is carried out at the middle of top plate (Figure 4)

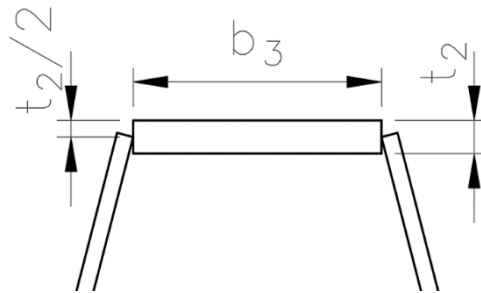


Figure 4. Fastening of webs to top plate

2. Projection of web thickness  $t'_3$  is equivalent to web thickness  $t_3$
3. Calculation for DR-type of Moment of Inertia for web without taking into account the displacement of gravity center relative to horizontal axis
4. Distance between the center of inclined web and the center of cross-section,  $z_w$ , is equal to distance between the center of vertical web and the center of cross-section,  $z_v$
5. Webs without holes are not significantly affecting the distance between the geometry center of cross section and the mass center.

The proof of the second assumption is described in APPENDIX 2.

For ease of reference the moment of inertia of the member cross-section is calculated after separate consideration of each member element. The estimation structure included the following items:

- Top plate
- Bottom plate
- Webs
- Downstands (if it is needed)
- Whole section

There are four variants of cross-section for each type of hollow steel section: without or with web holes and without downstands, with or without downstands and with web holes. Firstly, the member without downstands was analyzed. Secondly, the moments of inertia for cross-section with downstands were calculated. Reviewing those estimations it is possible to make conclusions about web holes and the combination of web holes and downstands and their influence to the steel member of Deltabeam®.

### 3.1 D-type

#### 3.1.1 Without web holes and downstands

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Web:  $t_3$
- Beam:  $h_1$ .

The cross-section for the calculations with the given data is shown in Figure 8.

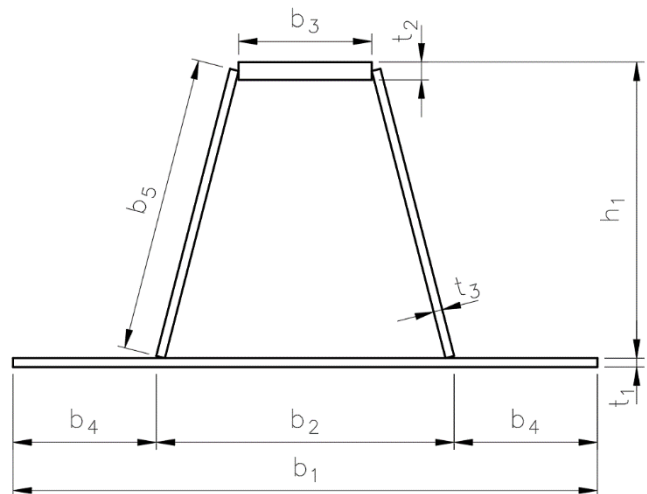


Figure 8. Cross-section for Deltabeam® without web holes and downstands

Parameters defining:

1. Top plate

Geometric layout for the calculations is shown in Figure 9 below:

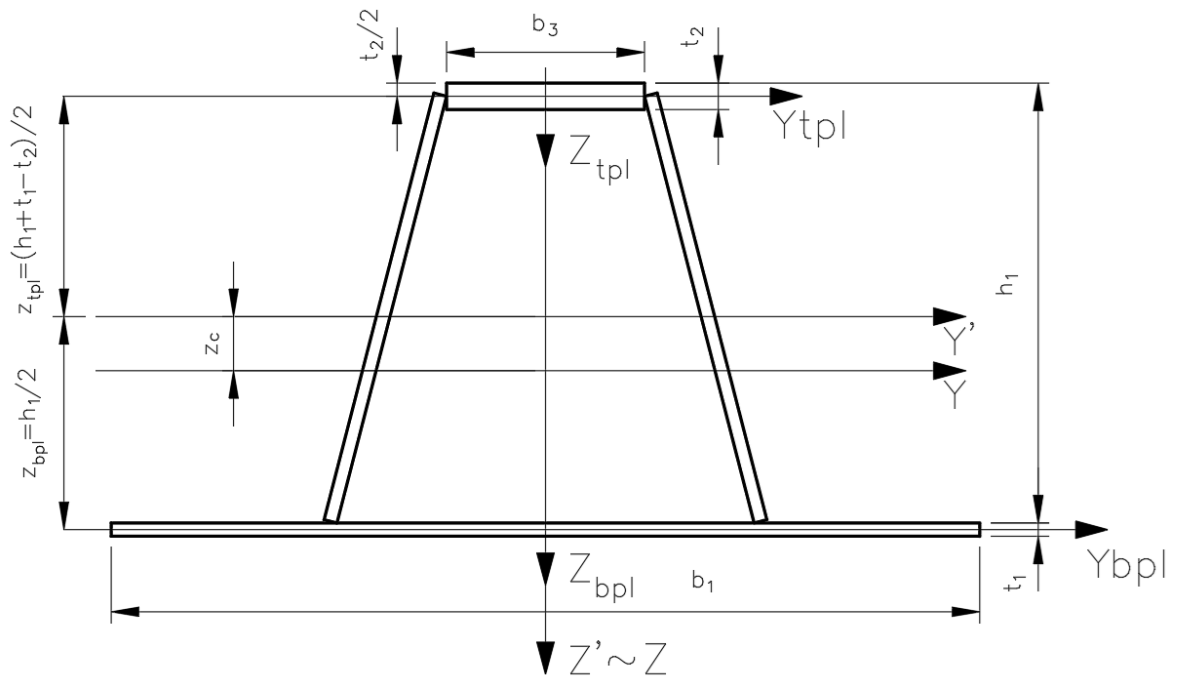


Figure 9. Geometric layout of cross-section with parameters for top and bottom plates

Where:

- $Y', Z'$  – axis passing through center of member cross-section
- $Y_{tpl}, Z_{tpl}, Y_{bpl}, Z_{bpl}$  – axis passing through center of member elements
- $Y, Z$  – central axis (neutral axis)
- $z_c$  – distance between center of cross-section and neutral axis (calculated as per formula (16) in 3.1.1)
- $z_{tpl}, z_{bpl}$  – distance between center of element and center of cross-section.

Formula for distance between center of top plate and center of cross-section is described below:

$$z_{tpl} = \frac{h_1 + t_1 - t_2}{2} \quad (4)$$

Moment of Inertia for top plate is calculated as per the formula:

$$I_{Y_{tpl}} = \frac{t_2 \cdot b_3^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 \quad (5)$$

2. Bottom plate

Geometric layout for calculations is shown in Figure 9.

The distance between the center of the bottom plate and the center of the cross-section is calculated as per the formula:

$$z_{bpl} = \frac{h_1}{2} \quad (6)$$

Moment of Inertia for bottom plate is calculated as per the formula:

$$I_{Y_{bpl}} = \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{tbl} - z_c)^2 \quad (7)$$

### 3. Webs

Geometric layout for the calculations is shown in Figure 10 below:

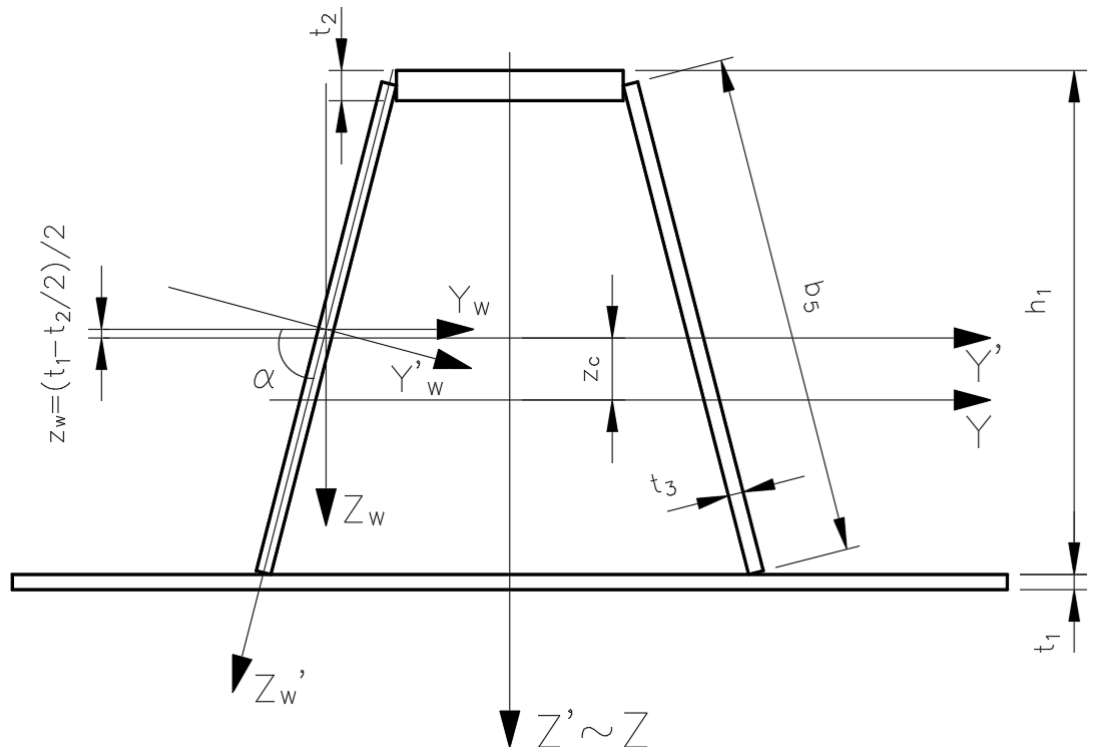


Figure 10. Geometric layout of cross-section with parameters for webs

Where:

- $Y', Z', Y'_w, Z'_w$  – axis passing through center of the cross-section
- $Y_w, Z_w$  – rotated figure axis for webs
- $Y, Z$  – central axis (neutral axis)
- $z_c$  – distance between center of cross-section and gravity center (calculated as per formula (16) in 3.1.1)
- $z_w$  – distance between center of element and center of cross-section.

Distance between center of top plate and center of cross-section is calculated as per the formula:

$$z_w = \frac{t_1 - \frac{t_2}{2}}{2} \quad (8)$$

$\sin\alpha$  and web width (APPENDIX 2, formula (1) and (2)):

$$\sin\alpha = \frac{h_1 - \frac{t_2}{2}}{b_5} \quad (1)$$

$$b_5 = \sqrt{\left(\frac{b_1 - b_3}{2} - b_4 - t_3\right)^2 + \left(h_1 - \frac{t_2}{2}\right)^2} \quad (2)$$

According to formula (1):  $\cos\alpha = \sqrt{1 - \sin^2\alpha}$  (1.1)

Moment of Inertia for web relative to  $Z'_w O Y'_w$ :

$$I_{Z'_w} = \frac{b_5 \cdot t_3^3}{12} \quad (9)$$

$$I_{Y'_w} = \frac{b_5^3 \cdot t_3}{12} \quad (10)$$

Moments of Inertia for web are calculated as per formulas:

$$I_{Z_w} = I_{Z'_w} \cdot \sin^2\alpha + I_{Y'_w} \cdot \cos^2\alpha \quad (11)$$

$$I_{Y_w} = I_{Z'_w} \cdot \cos^2\alpha + I_{Y'_w} \cdot \sin^2\alpha \quad (12)$$

Moment of Inertia for web relative to ZOY is calculated as per the formula:

$$I_{Y_{web}} = I_{Y_w} + t_3 \cdot b_5 \cdot (z_w + z_c)^2 \quad (13)$$

#### 4. Beam cross-section

Moment of Inertia for beam without web holes and downstands is calculated as per the formula:

$$I_Y = \sum I_{yi} = I_{Y_{tpl}} + I_{Y_{bpl}} + 2 \cdot I_{Y_{web}} \quad (14)$$

Apply (5), (7), (13) to (14):

$$I_{YD} = \frac{b_3 \cdot t_2^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 + \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{bpl} - z_c)^2 + 2 \cdot (I_{Y_w} + t_3 \cdot b_5 \cdot (z_w + z_c)^2) \quad (15),$$

where  $z_c$  is calculated as per the formula:

$$z_c = \frac{\sum S_y}{\sum A} = \frac{\frac{b_1 \cdot t_1 \cdot h_1}{2} - b_3 \cdot t_2 \cdot \frac{h_1 + t_1 - t_2}{2}}{b_1 \cdot t_1 + b_3 \cdot t_2 + 2 \cdot b_5 \cdot t_3} \quad (16)$$

### 3.1.2 With web holes and without downstands

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Web:  $t_3, d$
- Beam:  $h_1$ .

The cross-section for the calculations with the given data is shown in Figure 11.

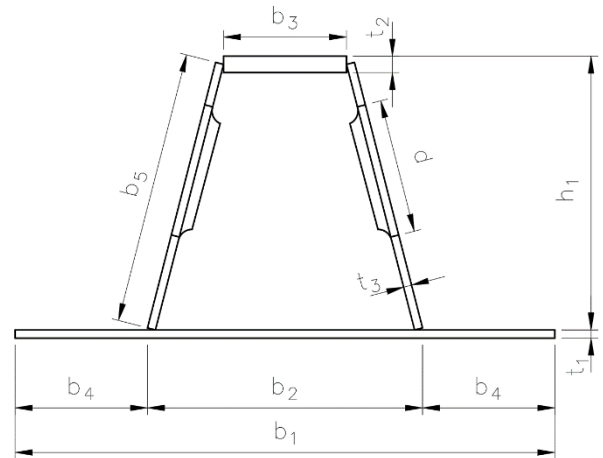


Figure 11. Cross-section for Deltabeam® with web holes and without downstands

Parameters defining:

1. Top plate

Calculations are the same as in 3.1.1 for the top plate.

2. Bottom plate

Calculations are the same as in 3.1.1 for the bottom plate.

### 3. Webs

The geometric layout for the calculations is shown in Figure 12 below:

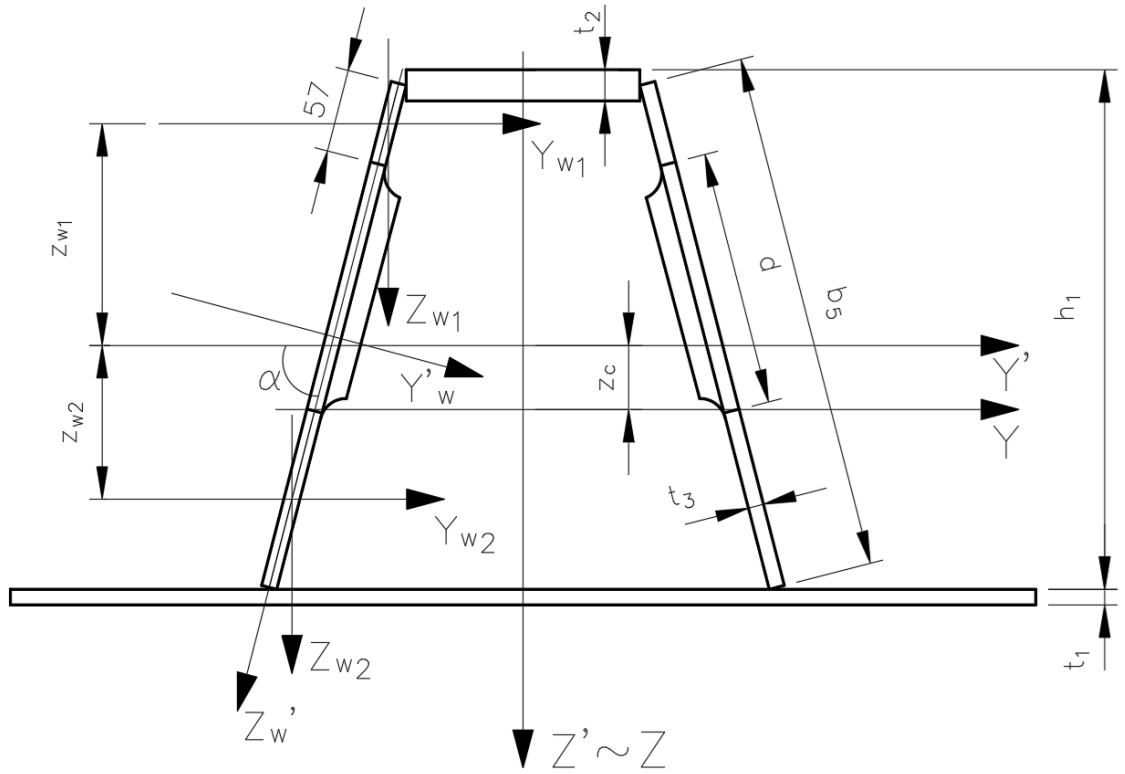


Figure 12. Geometric layout of cross-section with parameters for webs

Where:

- $Y', Z', Y'_w, Z'_w$  – axis passing through center of the cross-section
- $Y_{w1}, Z_{w1}, Y_{w2}, Z_{w2}$  – rotated figure axis for web parts
- $Y, Z$  – central axis (neutral axis)
- $Z_c$  – distance between center of cross-section and gravity center (calculated as per formula (25) in 3.1.2.)
- $Z_{w1}, Z_{w1}$  – distance between center of element and center of cross-section
- $d$  – hole diameter.

Distances between center of top plate and center of cross-section:

$$Z_{w1} = \frac{h_1 + t_1 - t_2 - 57 \sin \alpha}{2} \quad (17)$$

$$Z_{w2} = \frac{h_1 - t_1 - (b_5 - d - 57) \sin \alpha}{2} \quad (18)$$

$\sin \alpha$  and  $b_5$  are calculated as per formula (1) and (2) in APPENDIX 2.

$\cos \alpha$  is calculated as per the formula (1.1) in 3.1.1.

Moment of Inertia for web relative to  $Z_{wi}OY_{wi}$ :

$$I_{Z'_{w1}} = \frac{57 \cdot t_3^3}{12} \quad (19)$$

$$I_{Y'_{w1}} = \frac{57^3 \cdot t_3}{12} \quad (20)$$

$$I_{Z'_{w2}} = \frac{(b_5 - 57 - d) \cdot t_3^3}{12} \quad (21)$$

$$I_{Y'_{w2}} = \frac{(b_5 - 57 - d)^3 \cdot t_3}{12} \quad (22)$$

Moments of Inertia for web are calculated as per formulas (11) and (12) in 3.1.1.

Moment of Inertia for web with hole relative to ZOY is calculated as per the formula:

$$I_{Y_{web}} = I_{Y_{w1}} + I_{Y_{w2}} + t_3 \cdot 57 \cdot (z_{w1} + z_c)^2 + t_3 \cdot (b_5 - 57 - d) \cdot (z_{w2} - z_c)^2 \quad (23)$$

#### 4. Beam cross-section

Moment of Inertia for beam with web holes and without downstands is calculated as per the formula (14).

Apply (5), (7), (23) to (14):

$$I_{Y_{Dh}} = \frac{b_3 \cdot t_2^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 + \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{bpl} - z_c)^2 + 2 \cdot (I_{Y_{w1}} + I_{Y_{w2}} + t_3 \cdot 57 \cdot (z_{w1} + z_c)^2 + t_3 \cdot (b_5 - 57 - d) \cdot (z_{w2} - z_c)^2) \quad (24),$$

where  $z_c$  is calculated as per the formula:

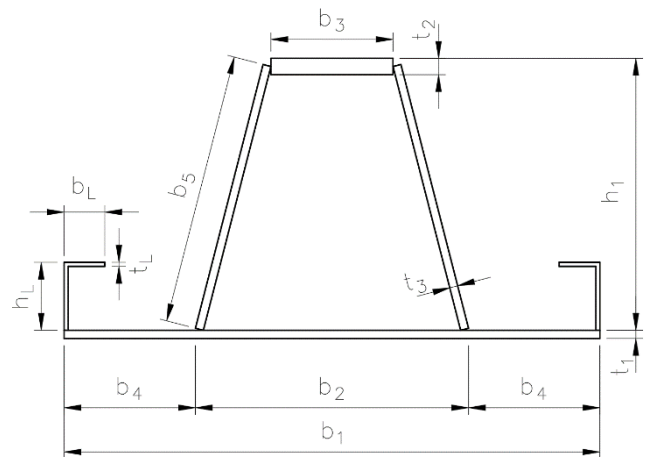
$$z_c = \frac{\sum S_y}{\sum A} = \frac{\frac{b_1 \cdot t_1 \cdot h_1}{2} - b_3 \cdot t_2 \cdot \frac{h_1 + t_1 - t_2}{2} - 2 \cdot (57 \cdot t_3 \cdot z_{w1} - (b_5 - 57 - d) \cdot t_3 \cdot z_{w2})}{b_1 \cdot t_1 + b_3 \cdot t_2 + 2 \cdot (b_5 - d) \cdot t_3} \quad (25)$$



### 3.1.3 With downstands and without web holes

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Web:  $t_3$
- Beam:  $h_1$
- Downstands:  $h_L, b_L, t_L$ .



The cross-section for the calculations with the given data is shown in Figure 13.

Figure 13. Cross-section for Deltabeam® with downstands and without web holes

Parameters defining:

Calculations for the top plate, the bottom plate and webs are the same as in 3.1.1.

#### 1. Downstands

Downstand is divided into two rectangles (Figure 14).

Rectangle №1 -  $h_L \times t_L$ ,

Rectangle №2 -  $(b_L - t_L) \times t_L$ .

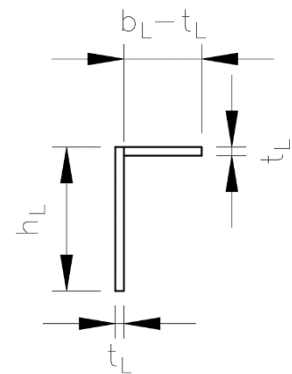


Figure 14. Downstand

Moment of Inertia for downstand parts relative to figure axis:

$$I_{YL_1} = \frac{t_L \cdot h_L^3}{12} \quad (26) \text{ – for rectangle №1,}$$

$$I_{YL_2} = \frac{(b_L - t_L) \cdot t_L^3}{12} \quad (27) \text{ – for rectangle №2.}$$

The geometric layout for the calculations is shown in Figure 15 below:

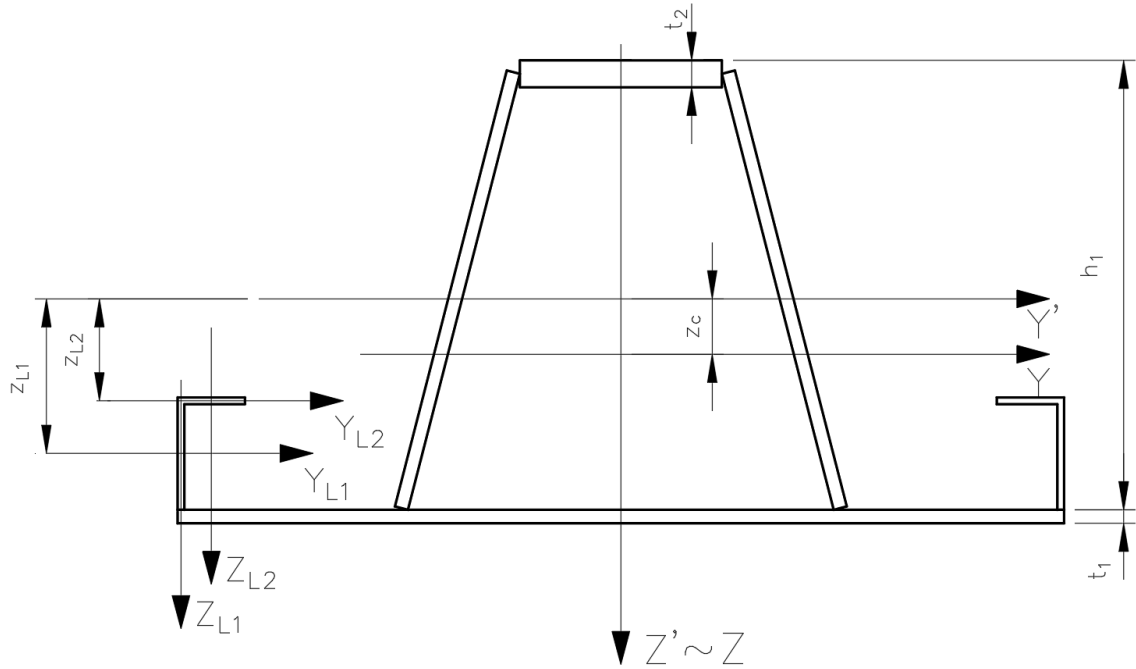


Figure 15. Geometric layout of cross-section with parameters for downstands

Where:

- $Y', Z', Y_{L1}, Z_{L1}, Y_{L2}, Z_{L2}$  – axis passing through center of the cross-section
- $Y, Z$  – central axis
- $z_c$  – distance between center of cross-section and neutral axis (calculated as per formula (33) in 3.1.3)
- $Z_{L1}, Z_{L2}$  – distance between center of element and center of cross-section.

Distances between center of top plate and center of cross-section:

$$z_{L1} = \frac{h_1 - t_1 - h_L}{2} \quad (28)$$

$$z_{L2} = \frac{h_1 - t_1 - 2 \cdot h_L + t_L}{2} \quad (29)$$

Moment of Inertia for downstands relative to ZOY is calculated as per formula:

$$I_{YL} = I_{Y_{L1}} + I_{Y_{L2}} + h_L \cdot t_L \cdot (z_{L1} - z_c)^2 + t_L \cdot (b_L - t_L) \cdot (z_{L2} - z_c)^2 \quad (30)$$





## 3.2 DR-type

### 3.2.1 Without web holes and downstand

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Inclined Web:  $t_3$
- Vertical web:  $t_3$
- Beam:  $h_1$ .

The cross-section for the calculations with the given data is shown in Figure 18.

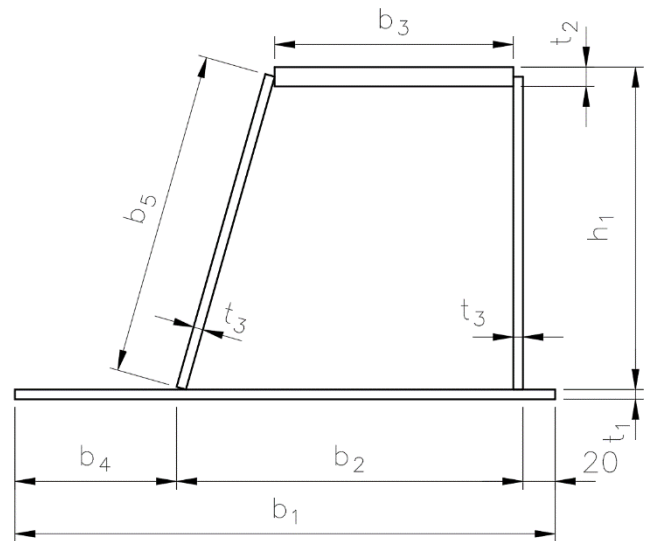


Figure 18. Cross-section for Deltabeam® without web holes and downstands

Parameters defining:

1. Development  $b_5$

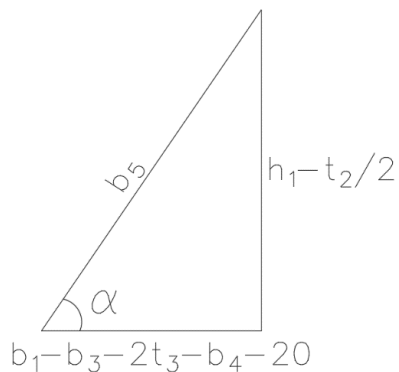


Figure 19. Geometric layout

According to Figure 19:

$\sin \alpha$  is calculated by the formula (1).

$$b_5 = \sqrt{(b_1 - b_3 - b_4 - 2 \cdot t_3 - 0,02)^2 + \left(h_1 - \frac{t_2}{2}\right)^2} \quad (37)$$



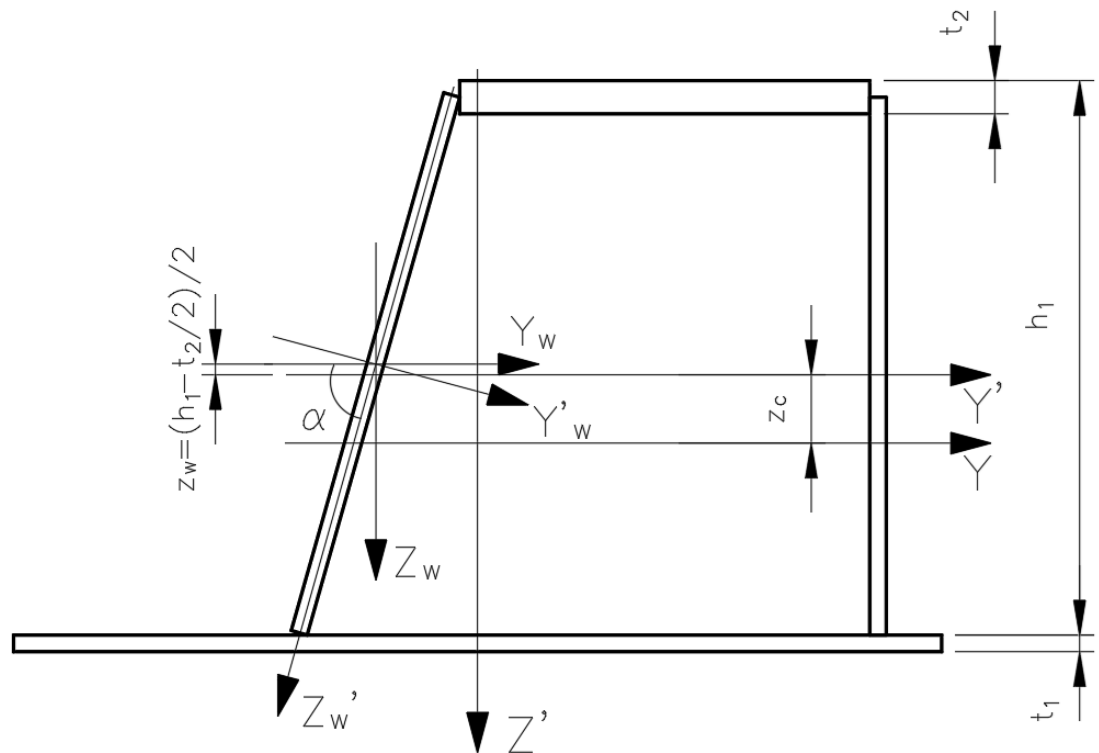


Figure 21. Geometric layout of cross-section with parameters for webs

Where:

- $Y', Z', Y'_w, Z'_w, Y_v, Z_v$  – axis passing through center of the cross-section
- $Y_w, Z_w$  – rotated figure axis for webs
- $Y$  – central axis
- $z_c$  – distance between center of cross-section and gravity center (calculated as per formula (42) in 3.2.1)
- $z_w, z_v$  – distance between center of element and center of cross-section.

4.1. Calculation for web are the same as in 3.1.1

4.2. Calculations for vertical web:

- $z_w$  is calculated by formula (8) in 3.1.1.
- $z_v = z_w$  (assumption 4)

Moment of Inertia for vertical web relative to  $Z_v O Y_v$ :

$$I_{Y_v} = \frac{(h_1 - \frac{t_2}{2})^3 \cdot t_3}{12} \quad (38)$$

Moment of Inertia for vertical web relative to  $Z O Y$  is calculated as per the formula:

$$I_{Y_{vweb}} = I_{Y_v} + t_3 \cdot (h_1 - \frac{t_2}{2}) \cdot (z_v + z_c)^2 \quad (39)$$

1. Beam cross-section

Moment of Inertia for beam without web holes and downstands is calculated as per the formula:

$$I_Y = \sum I_{yi} = I_{Y_{tpl}} + I_{Y_{bpl}} + I_{Y_{web}} + I_{Y_{vweb}} \quad (40)$$

Apply (5), (7), (13), (39) to (40):

$$I_{YDR} = \frac{b_3 \cdot t_2^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 + \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{bpl} - z_c)^2 + I_{Y_w} + t_3 \cdot b_5 \cdot (z_w + z_c)^2 + I_{Y_v} + t_3 \cdot (h_1 - \frac{t_2}{2}) \cdot (z_v + z_c)^2 \quad (41),$$

where  $z_c$  is calculated as per the formula:

$$z_c = \frac{\sum S_y}{\sum A} = \frac{\frac{b_1 \cdot t_1 \cdot h_1}{2} - b_3 \cdot t_2 \cdot \frac{h_1 + t_1 - t_2}{2}}{b_1 \cdot t_1 + b_3 \cdot t_2 + t_3 \cdot (b_5 + (h_1 - \frac{t_2}{2}))} \quad (42)$$

### 3.2.2 With web holes and without downstand

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Inclined web:  $t_3, d$
- Vertical web:  $t_3$
- Beam:  $h_1$ .

The cross-section for the calculations with the given data is shown in Figure 22.

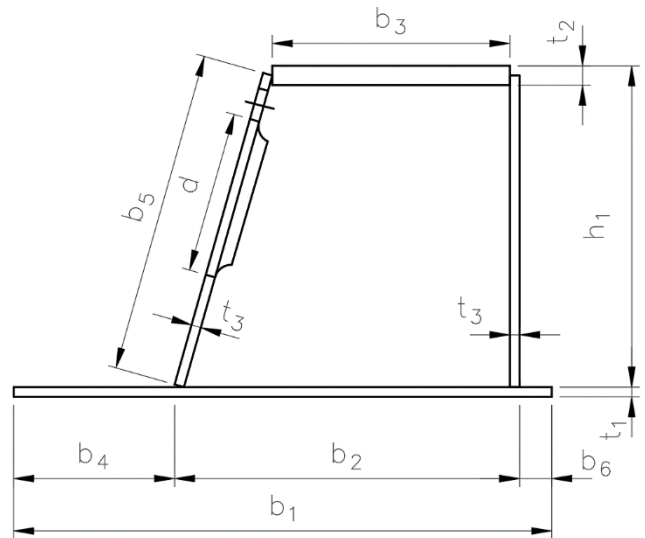


Figure 22. Cross-section for Deltabeam® with web holes and without downstands

Parameters defining:

Calculations for the top plate and the bottom plate are the same as in 3.1.1. Calculations for the inclined web are the same as in 3.1.2. Calculations for the vertical web are the same as in 3.2.1.



The geometric layout for the calculations is shown in Figure 23 below:

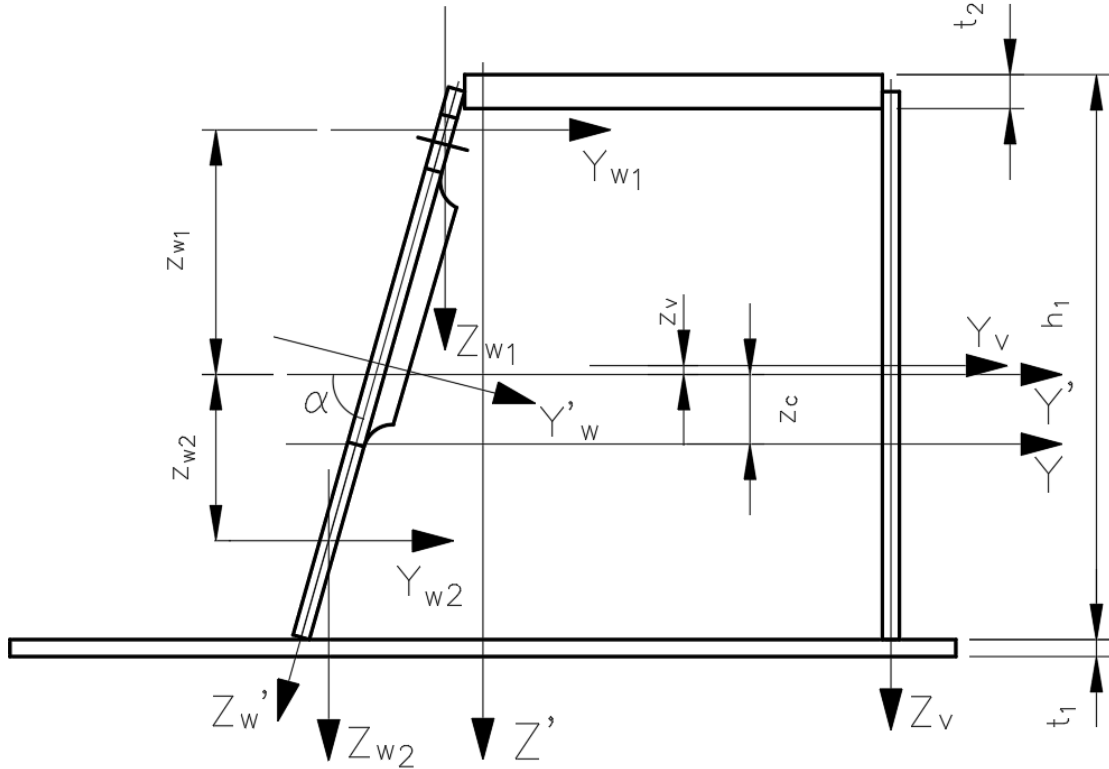


Figure 23. Geometric layout of cross-section with parameters for webs

Where:

- $Y', Z', Y'_{w1}, Z'_{w1}, Y'_{w2}, Z'_{w2}, Y_v, Z_v$  – axis passing through center of the cross-section
- $Y_w, Z_w$  – rotated figure axis for webs
- $Y$  – central axis
- $z_c$  – distance between center of cross-section and gravity center (calculated as per formula (44) in 3.2.2)
- $Z_w, Z_v$  – distance between center of element and center of cross-section
- $d$  - hole diameter.

Moment of Inertia for beam with web holes and without downstands is calculated as per the formula (40). Apply (5), (7), (23), (39) to (40):

$$I_{YDRh} = \frac{b_3 \cdot t_2^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 + \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{bpl} - z_c)^2 + I_{Y_{w1}} + I_{Y_{w2}} + t_3 \cdot 57 \cdot (z_{w1} + z_c)^2 + t_3 \cdot (b_5 - 57 - d) \cdot (z_{w2} - z_c)^2 + I_{Y_v} + t_3 \cdot (h_1 - \frac{t_2}{2}) \cdot (z_v + z_c)^2 \quad (43),$$

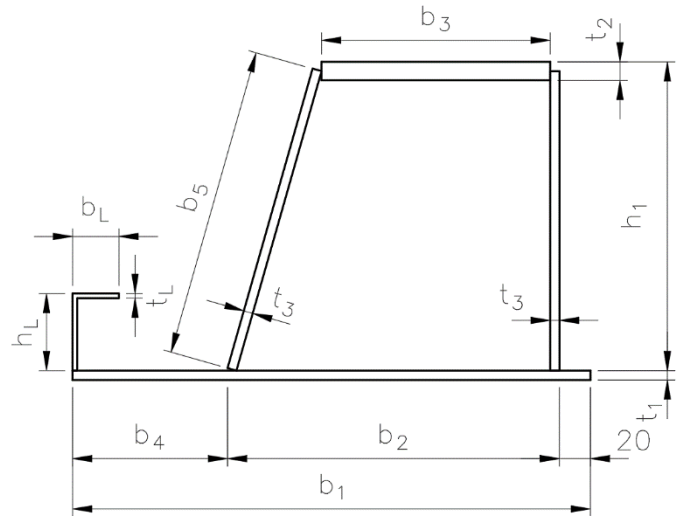
where  $z_c$  is calculated as per the formula:

$$z_c = \frac{\sum S_y}{\sum A} = \frac{\frac{b_1 \cdot t_1 \cdot h_1}{2} - b_3 \cdot t_2 \cdot \frac{h_1 + t_1 - t_2}{2} - 57 \cdot t_3 \cdot z_{w1} + (b_5 - 57 - d) \cdot t_3 \cdot z_{w2}}{b_1 \cdot t_1 + b_3 \cdot t_2 + t_3 \cdot (b_5 - d + (h_1 - \frac{t_2}{2}))} \quad (44)$$

### 3.2.3 Without web holes and with downstand

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Inclined web:  $t_3$
- Vertical web:  $t_3$
- Beam:  $h_1$
- Downstand:  $h_L, b_L, t_L$ .



The cross-section for the calculations with given data is shown in Figure 24.

Figure 24. Cross-section for Deltabeam® without web holes and with downstands

Parameters defining:

Calculations for the top plate, the bottom plate and the inclined web are the same as in 3.1.1. Calculations for the vertical web are the same as in 3.2.1. Calculations for downstand are the same as in 3.1.3. Geometric layout for calculations is shown in Figure 25 below:

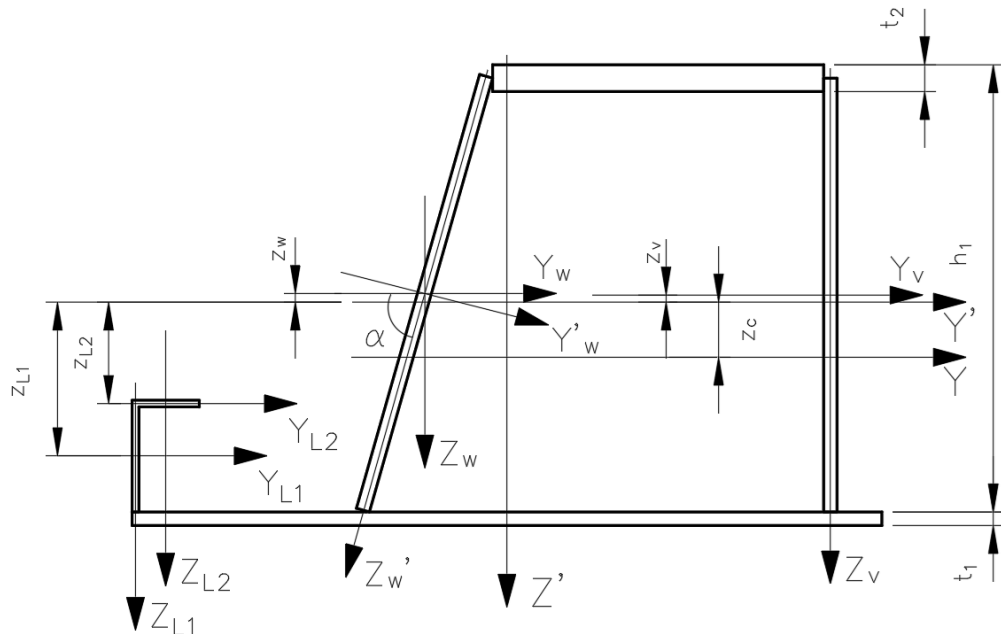


Figure 25. Geometric layout of cross-section with parameters for web holes and downstands

Where:

- $Y', Z', Y'_w, Z'_w, Y_{L1}, Z_{L1}, Y_{L2}, Z_{L2}, Y_v, Z_v$  – axis passing through center of the cross-section
- $Y_w, Z_w$  – rotated figure axis for webs
- $Y$  – central axis
- $z_c$  – distance between center of cross-section and gravity center (calculated as per formula (47) in 3.2.3)
- $Z_w, Z_v, Z_{L1}, Z_{L2}$  – distance between center of element and center of cross-section.

Moment of Inertia for beam with downstands and without web holes is calculated as per the formula:

$$I_Y = \sum I_{yi} = I_{Y_{tpl}} + I_{Y_{bpl}} + I_{Y_{web}} + I_{Y_{vweb}} + I_{Y_L} \quad (45)$$

Moment of Inertia for beam with web holes and without downstands is calculated as per formula (45). Apply (5), (7), (13), (30), (39) to (45):

$$I_{Y_{DR_L}} = \frac{b_3 \cdot t_2^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 + \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{bpl} - z_c)^2 + I_{Y_w} + t_3 \cdot b_5 \cdot (z_w + z_c)^2 + I_{Y_v} + t_3 \cdot \left(h_1 - \frac{t_2}{2}\right) \cdot (z_v + z_c)^2 + I_{Y_{L1}} + I_{Y_{L2}} + h_L \cdot t_L \cdot (z_{L1} - z_c)^2 + t_L \cdot (b_L - t_L) \cdot (z_{L2} - z_c)^2 \quad (46),$$

where  $z_c$  is calculated as per the formula:

$$z_c = \frac{\sum S_y}{\sum A} = \frac{\frac{b_1 \cdot t_1 \cdot h_1}{2} - b_3 \cdot t_2 \cdot \frac{h_1 + t_1 - t_2}{2} + h_L \cdot t_L \cdot \frac{h_1 - t_1 - h_L}{2} + t_L \cdot (b_L - t_L) \cdot \frac{h_1 - t_1 - 2 \cdot h_L + t_L}{2}}{b_1 \cdot t_1 + b_3 \cdot t_2 + t_3 \cdot \left(b_5 + \left(h_1 - \frac{t_2}{2}\right)\right) + t_L \cdot (h_L + b_L - t_L)} \quad (47)$$

### 3.2.4 With web holes and downstand

Given data:

- Top plate:  $t_2, b_3$
- Bottom plate:  $t_1, b_1, b_2, b_4$
- Inclined web:  $t_3, d$
- Vertical web:  $t_3$
- Beam:  $h_1$
- Downstand:  $h_L, b_L, t_L$ .

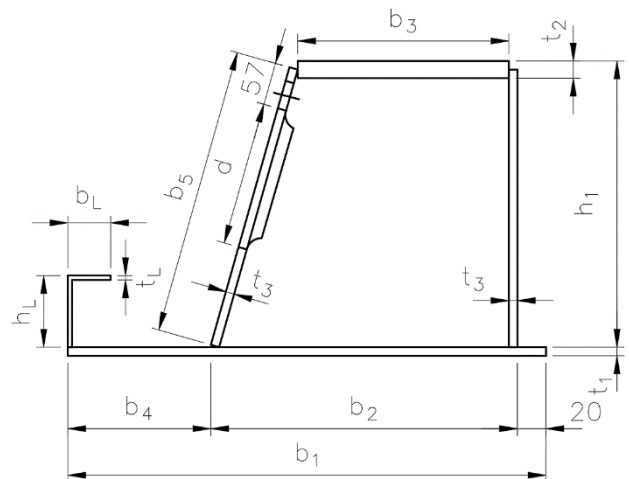


Figure 26. Cross-section for Deltabeam®

The cross-section for the calculations with the given data is shown in Figure 26.

Parameters defining:

Calculations for the top plate and the bottom plate are the same as in 3.1.1. Calculations for the inclined web are the same as in 3.1.2. Calculations for vertical web are the same as in 3.2.1. Calculations for downstand are the same as in 3.1.3. Geometric layout for calculations is shown in Figure 27 below:

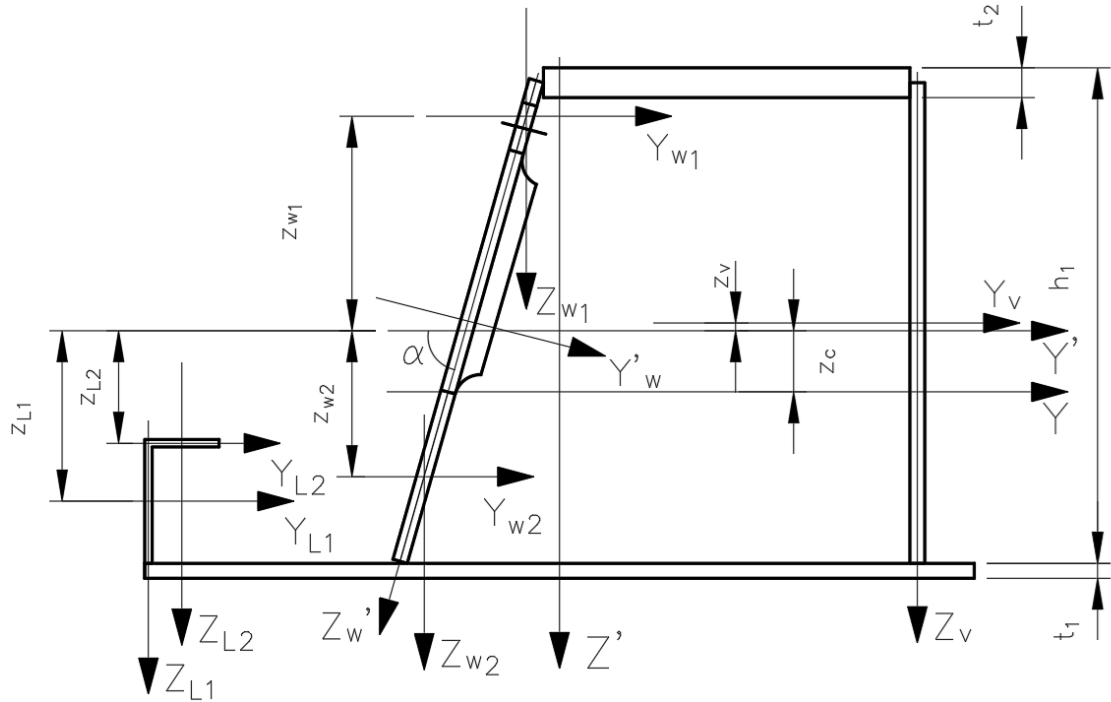


Figure 27. Geometric layout of cross-section with parameters for web holes and downstands

Where:

- $Y', Z', Y'_{w1}, Z'_{w1}, Y'_{w2}, Z'_{w2}, Y_{L1}, Z_{L1}, Y_{L2}, Z_{L2}, Y_v, Z_v$  – axis passing through center of the cross-section
- $Y_w, Z_w$  – rotated figure axis for webs
- $Y$  – central axis
- $Z_c$  – distance between center of cross-section and gravity center (calculated as per formula (49) in 3.2.4)
- $Z_{w1}, Z_{w2}, Z_v, Z_{L1}, Z_{L2}$  – distance between center of element and center of cross-section
- $d$  – hole diameter.

Moment of Inertia for beam with downstands and web holes is calculated as per formula (45).

Apply (5), (7), (23), (30), (39) to (45):

$$I_{YDR_{hL}} = \frac{b_3 \cdot t_2^3}{12} + t_2 \cdot b_3 \cdot (z_{tpl} + z_c)^2 + \frac{t_1^3 \cdot b_1}{12} + t_1 \cdot b_1 \cdot (z_{bpl} - z_c)^2 + I_{Y_{w1}} + I_{Y_{w2}} + t_3 \cdot 57 \cdot (z_{w1} + z_c)^2 + t_3 \cdot (b_5 - 57 - d) \cdot (z_{w2} - z_c)^2 + I_{Y_v} + t_3 \cdot \left(h_1 - \frac{t_2}{2}\right) \cdot (z_v + z_c)^2 + I_{Y_{L1}} + I_{Y_{L2}} + h_L \cdot t_L \cdot (z_{L1} - z_c)^2 + t_L \cdot (b_L - t_L) \cdot (z_{L2} - z_c)^2 \quad (48),$$

where  $z_c$  is calculated as per the formula:

$$z_c = \frac{\sum S_y}{\sum A} = \frac{\frac{b_1 \cdot t_1 \cdot h_1}{2} - b_3 \cdot t_2 \cdot \frac{h_1 + t_1 - t_2}{2} - 57 \cdot t_3 \cdot z_{w1} + (b_5 - 57 - d) \cdot t_3 \cdot z_{w2} + h_L \cdot t_L \cdot \frac{h_1 - t_1 - h_L}{2} + t_L \cdot (b_L - t_L) \cdot \frac{h_1 - t_1 - 2 \cdot h_L + t_L}{2}}{b_1 \cdot t_1 + b_3 \cdot t_2 + t_3 \cdot \left(b_5 - d + \left(h_1 - \frac{t_2}{2}\right)\right) + t_L \cdot (h_L + b_L - t_L)} \quad (49)$$

## 4 RESULTS

### 4.1 D-type

#### 4.1.1 Without/with web holes and without downstands

The calculation results for standard D-type beams are shown in Table 3.

Table 3

D-type	t1, mm	t2, mm	t3, mm	d, mm	$I_{YD}, cm^4$	$I_{YD_h}, cm^4$	$\Delta I_Y, cm^4$	$\varepsilon, \%$
D20-200	15	30	8	80	8561,6	8412,4	149	<b>1,74</b>
D20-300	15	30	8	80	12605,5	12513,3	92	<b>0,73</b>
D20-400	15	30	8	80	17771,3	17695,8	76	<b>0,42</b>
D22-300	15	30	8	80	15044,4	14875,7	169	<b>1,12</b>
D22-400	15	30	8	80	16975,5	16620,9	355	<b>2,09</b>
D25-300	15	30	8	150	18952,5	18477,7	475	<b>2,51</b>
D25-400	15	30	8	150	27366,4	26938,1	428	<b>1,57</b>
D26-300	15	30	8	150	21030,9	20443,6	587	<b>2,79</b>
D26-400	15	30	8	150	30379,6	29891	489	<b>1,61</b>
D30-300	15	30	8	150	25926,1	24805,7	1120	<b>4,32</b>
D30-400	15	30	8	150	38384,3	37567	817	<b>2,13</b>
D32-300	15	30	8	150	27687,7	25927,9	1760	<b>6,36</b>
D32-400	15	30	8	150	42169,5	40939,2	1230	<b>2,92</b>
D32-500	15	30	8	150	54198,8	53331,1	868	<b>1,60</b>
D37-400	15	30	8	150	53568,7	50872,6	2696	<b>5,03</b>
D37-500	15	30	8	150	70047,6	68133,8	1914	<b>2,73</b>
D40-400	15	30	8	150	63459,7	59873,7	3586	<b>5,65</b>
D40-500	15	30	8	150	82723,3	80123,7	2600	<b>3,14</b>
D40-600	15	30	8	150	101092,8	99090,3	2003	<b>1,98</b>
D50-500	15	30	8	150	122362,9	115222,5	7140	<b>5,84</b>
D50-600	15	30	8	150	152461,7	146825,9	5636	<b>3,70</b>
							max	<b>6,36</b>

Engineering calculations lead to the conclusion that web holes affected to the beam bending stiffness and it means that they need to be taken into account. Web holes reduce the moment of inertia values.

#### 4.1.2 With web holes and with/without downstands

The calculation results for standard D-type beams are shown in Table 4. For this estimation downstands with dimensions 50x30x3 were adopted.

Table 4

D-type	t1, mm	t2, mm	t3, mm	d, mm	$I_{YD_h}, cm^4$	$I_{YD_{hL}}, cm^4$	$\Delta I_Y, cm^4$	$\varepsilon, \%$
D20-200	15	30	8	80	8412,4	8486,7	-74	<b>0,88</b>
D20-300	15	30	8	80	12513,3	12641,1	-128	<b>1,02</b>
D20-400	15	30	8	80	17695,8	17853,2	-157	<b>0,89</b>
D22-300	15	30	8	80	14875,7	15031,8	-156	<b>1,05</b>
D22-400	15	30	8	80	16620,9	16717,6	-97	<b>0,58</b>
D25-300	15	30	8	150	18477,7	18686,9	-209	<b>1,13</b>
D25-400	15	30	8	150	26938,1	27213,9	-276	<b>1,02</b>
D26-300	15	30	8	150	20443,6	20670,1	-227	<b>1,11</b>
D26-400	15	30	8	150	29891	30192,8	-302	<b>1,01</b>
D30-300	15	30	8	150	24805,7	25065,3	-260	<b>1,05</b>
D30-400	15	30	8	150	37567	37945,0	-378	<b>1,01</b>
D32-300	15	30	8	150	25927,9	26173,1	-245	<b>0,95</b>
D32-400	15	30	8	150	40939,2	41332,5	-393	<b>0,96</b>
D32-500	15	30	8	150	53331,1	53875,4	-544	<b>1,02</b>
D37-400	15	30	8	150	50872,6	51339,5	-467	<b>0,92</b>
D37-500	15	30	8	150	68133,8	68811,3	-678	<b>0,99</b>
D40-400	15	30	8	150	59873,7	60446,9	-573	<b>0,96</b>
D40-500	15	30	8	150	80123,7	80943,7	-820	<b>1,02</b>
D40-600	15	30	8	150	99090,3	100111,9	-1022	<b>1,03</b>
D50-500	15	30	8	150	115223	116365,3	-1143	<b>0,99</b>
D50-600	15	30	8	150	146826	148311,4	-1486	<b>1,01</b>
							max	<b>1,13</b>

Engineering calculations lead to the conclusion that influence of downstands to bending stiffness of the beam is inconspicuous. In all cases downstands increase moment of inertia values.

### 4.1.3 Results comparison for beams with web holes and various downstands

The first step of calculation is estimations with the minimum dimensions for downstands. The second step is to get results for the maximum dimensions. The results and data for the second part are shown in Table 5, where  $I_{YD_{hL}}$  is moment of inertia for beams with downstands with minimum dimensions (50x30x3) and  $I_{YD_{hLL}}$  is moment of inertia for beams with maximum values for downstands (shown in Table 5).

Table 5

D-type	hl, mm	bl, mm	tl, mm	d, mm	$I_{YD_{hLL}}, cm^4$	$I_{YD_{hL}}, cm^4$	$\Delta I_Y, cm^4$	$\varepsilon, \%$
D20-200	80	60	5	80	8508,7	8486,7	22	<b>0,26</b>
D20-300	80	60	5	80	12654,7	12641,1	14	<b>0,11</b>
D20-400	80	60	5	80	17872,4	17853,2	19	<b>0,11</b>
D22-300	80	60	5	80	15050,2	15031,8	18	<b>0,12</b>
D22-400	80	60	5	80	16732,4	16717,6	15	<b>0,09</b>
D25-300	120	60	6	150	18828,5	18686,9	142	<b>0,75</b>
D25-400	120	60	6	150	27358,2	27213,9	144	<b>0,53</b>
D26-300	120	60	6	150	20809,4	20670,1	139	<b>0,67</b>
D26-400	120	60	6	150	30345,3	30192,8	153	<b>0,50</b>
D30-300	140	60	6	150	25311,4	25065,3	246	<b>0,97</b>
D30-400	140	60	6	150	38188,8	37945,0	244	<b>0,64</b>
D32-300	140	60	6	150	26425,7	26173,1	253	<b>0,96</b>
D32-400	140	60	6	150	41581,3	41332,5	249	<b>0,60</b>
D32-500	140	60	6	150	54223	53875,4	348	<b>0,64</b>
D37-400	180	70	6	150	51920,2	51339,5	581	<b>1,12</b>
D37-500	180	70	6	150	69419,3	68811,3	608	<b>0,88</b>
D40-400	200	70	6	150	61261,2	60446,9	814	<b>1,33</b>
D40-500	200	70	6	150	81793,8	80943,7	850	<b>1,04</b>
D40-600	200	70	6	150	101132,7	100111,9	1021	<b>1,01</b>
D50-500	200	70	6	150	117520,2	116365,3	1155	<b>0,98</b>
D50-600	200	70	6	150	150010,9	148311,4	1700	<b>1,13</b>
							max	<b>1,33</b>

After reviewing the estimation result, it could be concluded that larger downstands affect more on beam stiffness.



## 4.2 DR-type

### 4.2.1 Without/with web holes and without downstands

The calculation results for standard DR-type beams are shown in Table 6.

Table 6

DR-type	t1, mm	t2, mm	t3, mm	d, mm	$I_{YDR}, cm^4$	$I_{YDRh}, cm^4$	$\Delta I_Y, cm^4$	$\varepsilon, \%$
DR20-215	15	30	8	80	9725,2	9692,2	33	<b>0,34</b>
DR20-245	15	30	8	80	11037	11005,3	32	<b>0,29</b>
DR22-250	15	30	8	80	13641,9	13602,1	40	<b>0,29</b>
DR25-260	15	30	8	150	18205,8	17991,2	215	<b>1,18</b>
DR26-230	15	30	8	150	18279,4	18065,8	214	<b>1,17</b>
DR26-260	15	30	8	150	20647	20436,3	211	<b>1,02</b>
DR26-290	15	30	8	150	22867	22655,1	212	<b>0,93</b>
DR26-325	15	30	8	150	25414,9	25198,8	216	<b>0,85</b>
DR30-270	15	30	8	150	27341,5	27082,5	259	<b>0,95</b>
DR32-250	15	30	8	150	28295,2	27907,9	387	<b>1,37</b>
DR32-285	15	30	8	150	32011,2	31667	344	<b>1,08</b>
DR32-310	15	30	8	150	35132	34828,2	304	<b>0,86</b>
DR32-265	15	30	8	150	39737,7	39437,1	301	<b>0,76</b>
DR37-325	15	30	8	150	50318,6	49671,8	647	<b>1,29</b>
DR40-295	15	30	8	150	54238,1	53241,5	997	<b>1,84</b>
DR50-350	15	30	8	150	99157,9	96868,1	2290	<b>2,31</b>
							max	<b>2,31</b>

Engineering calculations lead to the conclusion that web holes affected the beam bending stiffness and they need to be taken into account.

#### 4.2.2 With web holes and with/without downstands

The calculation results for standard DR-type beams are shown in Table 7. For this estimation downstands with dimensions 50x30x3 were adopted.

Table 7

DR-type	t1, mm	t2, mm	t3, mm	d, mm	$I_{YDR_h}, cm^4$	$I_{YDR_hL}, cm^4$	$\Delta I_Y, cm^4$	$\varepsilon, \%$
DR20-215	15	30	8	80	9692,2	9699,1	-7	<b>0,07</b>
DR20-245	15	30	8	80	11005,3	11011,9	-7	<b>0,06</b>
DR22-250	15	30	8	80	13602,1	13610,3	-8	<b>0,06</b>
DR25-260	15	30	8	150	17991,2	18002,5	-11	<b>0,06</b>
DR26-230	15	30	8	150	18065,8	18079	-13	<b>0,07</b>
DR26-260	15	30	8	150	20436,3	20449,1	-13	<b>0,06</b>
DR26-290	15	30	8	150	22655,1	22667,4	-12	<b>0,05</b>
DR26-325	15	30	8	150	25198,8	25210,5	-12	<b>0,05</b>
DR30-270	15	30	8	150	27082,5	27098,6	-16	<b>0,06</b>
DR32-250	15	30	8	150	27907,9	27926,1	-18	<b>0,07</b>
DR32-285	15	30	8	150	31667	31684,6	-18	<b>0,06</b>
DR32-310	15	30	8	150	34828,2	34845,4	-17	<b>0,05</b>
DR32-265	15	30	8	150	39437,1	39452,8	-16	<b>0,04</b>
DR37-325	15	30	8	150	49671,8	49692,5	-21	<b>0,04</b>
DR40-295	15	30	8	150	53241,5	53266,2	-25	<b>0,05</b>
DR50-350	15	30	8	150	96868,1	96903,3	-35	<b>0,04</b>
							max	<b>0,07</b>

Engineering calculations lead to the conclusion that the influence of downstands to the beam bending is inconspicuous.

### 4.2.3 Results comparison for beams with web holes and various downstands

The calculation results are shown in Table 8, where  $I_{YDR_{hL}}$  is moment of inertia for beams with downstands with minimum dimensions (50x30x3) and  $I_{YDR_{hLL}}$  is moment of inertia for beams with maximum values for downstands (shown in Table 8).

Table 8

DR-type	hl, mm	bl, mm	tl, mm	d, mm	$I_{YDR_{hLL}}, cm^4$	$I_{YDR_{hL}}, cm^4$	$\Delta I_Y, cm^4$	$\varepsilon, \%$
DR20-215	80	60	5	80	9702,3	9699,1	3	<b>0,03</b>
DR20-245	80	60	5	80	11016	11011,9	4	<b>0,04</b>
DR22-250	80	60	5	80	13617,5	13610,3	7	<b>0,05</b>
DR25-260	120	60	6	150	18013,5	18002,5	11	<b>0,06</b>
DR26-230	120	60	6	150	18092,3	18079	13	<b>0,07</b>
DR26-260	120	60	6	150	20465,6	20449,1	17	<b>0,08</b>
DR26-290	120	60	6	150	22685,6	22667,4	18	<b>0,08</b>
DR26-325	120	60	6	150	25230,1	25210,5	20	<b>0,08</b>
DR30-270	140	60	6	150	27121,5	27098,6	23	<b>0,08</b>
DR32-250	140	60	6	150	27952,3	27926,1	26	<b>0,09</b>
DR32-285	140	60	6	150	31714,9	31684,6	30	<b>0,10</b>
DR32-310	140	60	6	150	34879,8	34845,4	34	<b>0,10</b>
DR32-265	140	60	6	150	39486,2	39452,8	33	<b>0,08</b>
DR37-325	180	70	6	150	49729,6	49692,5	37	<b>0,07</b>
DR40-295	200	70	6	150	53305,8	53266,2	40	<b>0,07</b>
DR50-350	200	70	6	150	97043,7	96903,3	140	<b>0,14</b>
							max	<b>0,14</b>

After reviewing the estimation result, it could be concluded that larger downstands affect more on beam stiffness.

## 5 CONCLUSIONS

According to the calculation results the influence of downstands is inconspicuous for all variants of cross-section for each Deltabeam® type, that is, downstands are not able to affect the bending stiffness of steel member, so, the contribution of downstands to bending stiffness of the steel member is not enough to affect significantly to the deflection.

There are some factors that may affect the deflection of beam at the erection stage and should be analyzed:

- 1 Impact on executed precamber
  - 1.1 Welding heat of downstands
  - 1.2 Welding quality
  - 1.3 Welding material (filler material)
- 2 Installation
  - 2.1 Installation quality
  - 2.2 Influence of equipment
- 3 Physico-mechanical properties of steel
- 4 The combination of factors

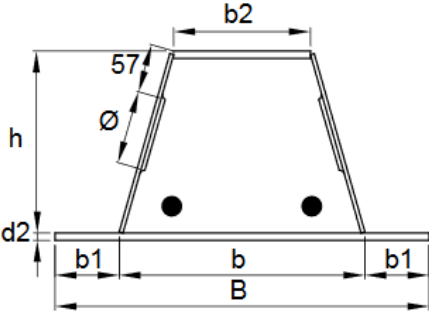
The next most likely reason that affects the deflection is impact on executed precamber. The reason of divergence in results might be the welding heat of downstands that could change the real value of executed precamber, consequently, the real value of deflection. It should be considered more closely, also in conjunction with downstand influence that was described in this research.

## REFERENCES

1. Peikko Group, 2014. DELTABEAM – Slim Floor Structures, Technical Manual.
2. Vartianen, M. 2018. DELTABEAM Design Handbook.

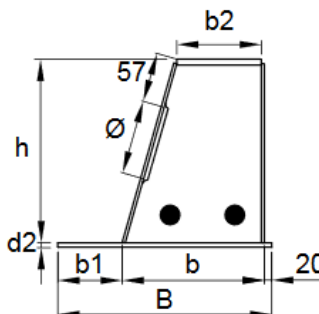
Deltabeam® dimensions are shown in Table 1 and Table 2.

Table1. The standard D-type Deltabeam® profiles (1)



b	B	b1*	b2	d2	h	Ø**
[mm]						
D20-200	395	97,5	100	5-25	200	80
D20-300	495	97,5	180	5 - 25	200	80
D20-400	660	130	278	5 - 25	200	80
D22-300	495	97,5	170	5 - 25	220	80
D22-400	660	130	270	5 - 25	220	80
D25-300	495	97,5	155	5 - 25	250	150
D25-400	660	130	255	5 - 25	250	150
D26-300	495	97,5	148	5 - 25	265	150
D26-400	660	130	245	5 - 25	265	150
D30-300	495	97,5	130	5 - 25	300	150
D30-400	660	130	230	5 - 25	300	150
D32-300	495	97,5	110	5 - 25	320	150
D32-400	660	130	210	5 - 25	320	150
D37-400	660	130	180	5 - 25	370	150
D37-500	760	130	278	5 - 25	370	150
D40-400	660	130	180	5 - 25	400	150
D40-500	760	130	278	5 - 25	400	150
D50-500	760	130	230	5 - 25	500	150
D50-600	860	130	330	5 - 25	500	150

Table 2. The standard DR-type Deltabeam® profiles (1)



b	B	b1*	b2	d2	h	Ø**
[mm]						
DR20-215	335	100	148	5 - 25	200	80
DR20-245	365	100	180	5 - 25	200	80
DR22-250	370	100	180	5 - 25	220	80
DR25-260	380	100	180	5 - 25	250	150
DR26-230	350	100	148	5 - 25	265	150
DR26-260	380	100	180	5 - 25	265	150
DR26-290	410	100	210	5 - 25	265	150
DR26-325	445	100	245	5 - 25	265	150
DR30-270	390	100	180	5 - 25	300	150
DR32-250	370	100	148	5 - 25	320	150
DR32-285	405	100	180	5 - 25	320	150
DR32-310	430	100	210	5 - 25	320	150
DR32-365	485	100	245	5 - 25	320	150
DR37-325	475	130	210	5 - 25	370	150
DR40-295	445	130	180	5 - 25	400	150
DR50-350	500	130	210	5 - 25	500	150

Given data:

- $b_1, b_2, b_3, b_4, t_1, t_2, t_3, h_1,$
- beam cross-section (Figure 5).

Aim: proof the equivalence of  $t_3$  and  $t_3'$  .

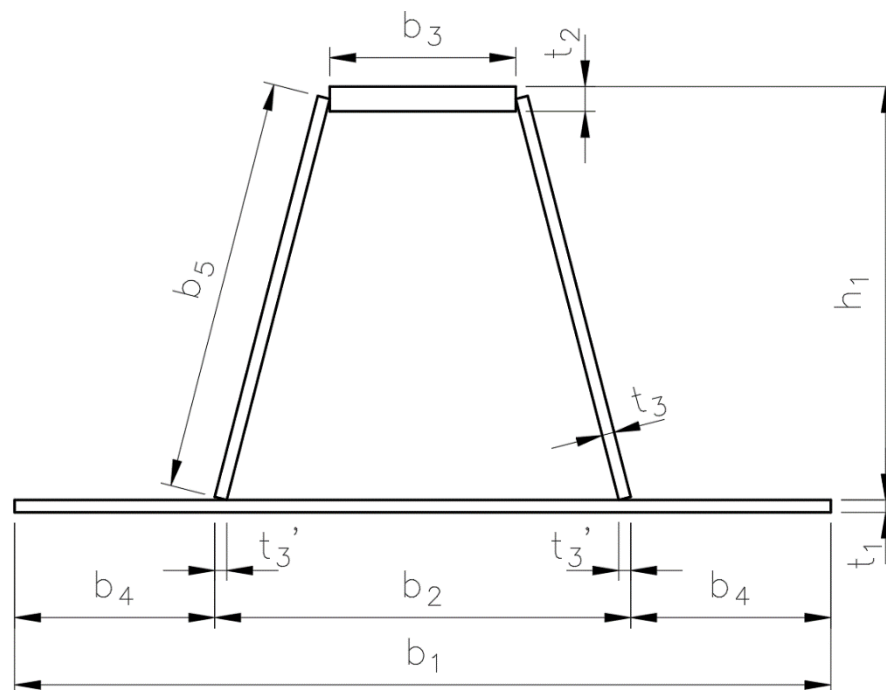


Figure 5. Deltabeam® cross-section

1. Development  $b_5$

According to Figure 6:

$$\sin \alpha = \frac{h_1 - \frac{t_2}{2}}{b_5} \quad (1)$$

$$b_5 = \sqrt{\left(\frac{b_1 - b_3}{2} - b_4 - t_3\right)^2 + \left(h_1 - \frac{t_2}{2}\right)^2} \quad (2)$$

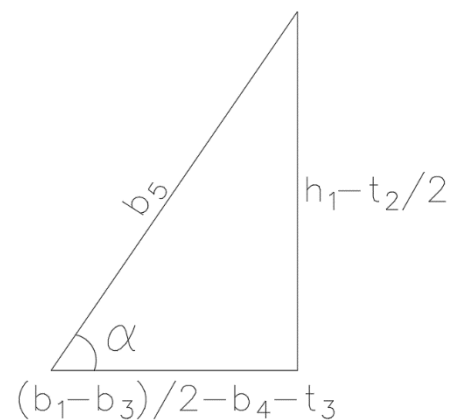


Figure 6. Geometric layout



2(2)

According to Figure 7:

$$t'_3 = t_3 \cdot \sin\alpha \quad (3)$$

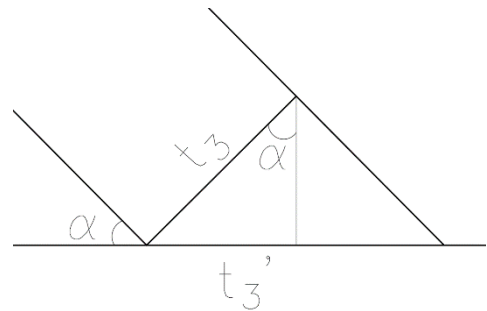


Figure 7. Geometric layout

## 2. Calculations for D70-900

$$\text{According to (2): } b_5 = \sqrt{\left(\frac{1100-580}{2} - 100 - 8\right)^2 + \left(700 - \frac{30}{2}\right)^2} \approx 703,438 \text{ mm}$$

$$\text{According to (1): } \sin\alpha = \frac{700 - \frac{30}{2}}{703,438} \approx 0,974$$

$$t_3 = 8 \text{ mm, according to (3): } t'_3 = 8 \cdot 0,974 = 7,792 \text{ mm}$$

$$\Delta = t_3 - t'_3 = 8 - 7,792 = 0,208 \text{ mm}$$

$$\mathcal{E} = \frac{\Delta}{t_3} \cdot 100\% = \frac{0,208}{8} \cdot 100\% = 2,6\%$$

## 3. Conclusion

$\Delta \ll 0$  and  $\mathcal{E} = 2,6\%$ , consequently,  $t'_3 = t_3$ .