

Saimaa University of Applied Sciences  
Technology, Lappeenranta  
Double Degree Programme in Civil and Construction Engineering

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# **Design of dynamically loaded foundations for pumps and compressors**

Bachelor's Thesis 2019

## **Abstract**

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The work was commissioned by Neste Engineering Solutions. The purpose of the work was to study the design process of dynamically loaded foundations. The process of calculating foundations for vibrating machines is long and complex, it includes many factors that should be taken into account. That is why the task was to create a manual and the Excel file for fast calculation.

Data for this study were collected by the mentor, it included compressor drawings and dynamic parameters of the machine. The information for the design was gathered from literature and articles taken from the Internet.

The final result of this thesis was an Excel file for calculating two different types of vibrating machines: Rotary and Reciprocating. This file allows to check the designed foundation for compliance with the main criteria required for the dynamically loaded foundations.

Keywords: foundation, dynamic loads, resonance, amplitude, vibrations, frequency, damping

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## Symbols

A – cross-section as area of pile

B - Width of rectangular foundation block, m

$B_z, B_x, B_\psi, B_\theta$  - Mass ratio; vertical, horizontal, rocking, pitching modes

E- modulus of elasticity of pile

F- Excitation force, kN

$f_{res}$  – Resonant frequency, Hz

G – Shear modulus of soil, kN/m<sup>2</sup>

g – Acceleration of gravity= 9,81 m/sec<sup>2</sup>

h - Effective foundation embedment depth, m

I – moment of inertia of pile about axis to resist displacement

$I_\psi, I_\theta$  - Mass moment of inertia in the rocking and pitching directions, kN\*m\*sec<sup>2</sup>

J- torsion moment of inertia of pile

$k_z, k_x, k_\psi, k_\theta$  - Spring constant; vertical, horizontal, rocking, pitching modes, kN/m

L – length of rectangular foundation block, m

$L_p$  – length of pile, m

M – Dynamic magnification factor

m – Mass, kN\*sec<sup>2</sup>/m

$n_\psi, n_\theta$  - Rocking and pitching mass ratio factors for geometric damping

$P_H$  – Force transmitted in the horizontal direction, kN

$P_V$  – Force transmitted in the vertical direction, kN

$P_{T\psi}$  – Transmitted rocking moment, kN\*m

$P_{T\theta}$  – Transmitted pitching moment, kN\*m

$p_{dyn}$  – Bearing pressure due to transmitted dynamic force, kN/m<sup>2</sup>

r – Ratio of operating frequency to natural frequency

$r_0$  – Equivalent radius for rectangular footing, m

$R_{hx}$  – Horizontal distance from center to edge of footing in the x-direction, m

$R_{hy}$  – Horizontal distance from center to edge of footing in the y-direction, m

$R_v$  – Vertical distance from base to horizontal machine load, m

$S_{all}$  – Allowable soil pressure, kN/m<sup>2</sup>

$T_1$  – Transmissibility factor for primary operating frequency

$T_2$  – Transmissibility factor for secondary operating frequency

$V_p, V_s$  – Shear wave velocities in pile and soil respectively

$W$  – Total weight of machine plus foundation, kN

$X$  – Displacement response in the horizontal x-direction, mm

$X_t$  – Total displacement response in the horizontal x-direction, mm

$Z$  – Displacement response in the vertical z-direction, mm

$Z_t$  – Total displacement response in the vertical z-direction, mm

$\alpha_z, \alpha_x, \alpha_\psi, \alpha_\theta$  - Damping ratio embedment factor; vertical, horizontal, rocking, pitching modes

$\mu$  – Poisson's ratio of soil

$\rho, \gamma_s$  – density of soil, kN/m<sup>3</sup>

$\xi_z, \xi_x, \xi_\psi, \xi_\theta$  (or  $c_z, c_x, c_\psi, c_\theta$ ) – Geometric damping ratios; vertical, horizontal, rocking, pitching modes

$\omega$  – Operating speed of machine, rpm

# **1 Introduction**

While designing foundations for machines, there is usually a question about taking into account dynamic loads, which directly affect the calculation and design. It also raises the question of choosing the right constructive solution to the foundation in which his work under the influence of a dynamic load will not have undesirable phenomena associated with his vibrations. When designing a foundation for machinery, the designation of its dimensions, the choice of concrete and reinforcement require careful attention. The engineer must always solve his problem in a complex. He acts not only as a calculator who, on the basis of a dynamic calculation, should give a correct forecast of the expected level of fluctuations of the foundation. He needs to ensure that oscillations are permissible, do not exceed certain limits. The amplitude of vibration of a machine at its operating frequency is the most important parameter to be determined in designing a machine foundation, in addition to the natural frequency of a machine foundation soil system. [3] Guidelines for the designing of foundations will be presented.

## **1.1 Types of machines**

When designing foundations for machinery and equipment, the primary issue is the need to take into account the calculations of dynamic loads. All machines, without exception, transmit vibrations of varying intensity to the bases, however, when some machines operate, considerable inertia forces arise, while the operation of other machines is characterized by a very small level of dynamic effects. In this connection, dynamic calculations of bases and foundations will have to be made for machines of rotary (high-speed machines like turbogenerators or rotary compressors), reciprocating (the machines that produce periodic unbalanced forces, such as steam engines), and impact (these machines produce impact loads, for instance, forging hammers) effects.

## 1.2 Types of foundations

By design, the foundations for machines can be block, wall and box type. Block foundations are made in the form of blocks and slabs. The box foundations consist of rigid walls connecting the upper and lower plates. Wall foundations are formed by columns embedded in the lower slab, with the upper parts of the columns being joined by beams or slabs. From the point of view of the work of structures, block and box foundations are the foundations of a rigid type, and the wall foundations are of an elastic type. Examples of constructive solutions for block, wall and box foundations are shown in Figure 1. [3]

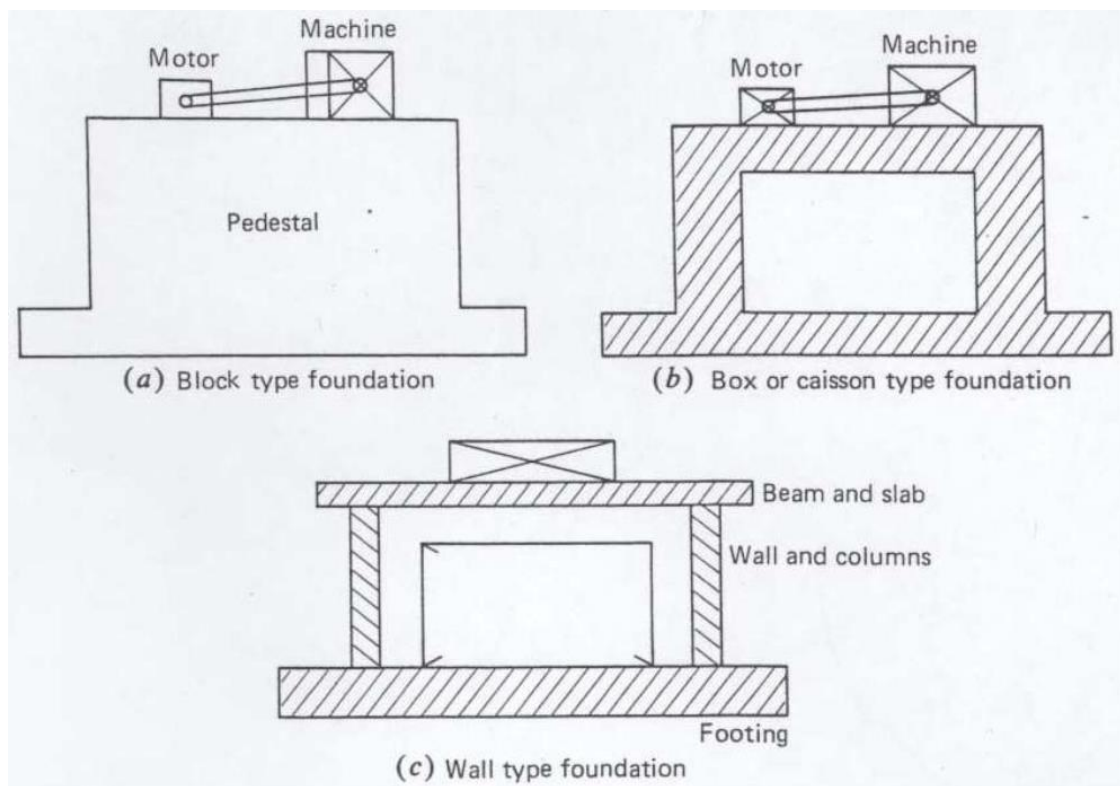


Figure 1. Foundation types (a) block type foundation, (b) box or caisson type foundation, (c) wall type foundation [3]

## 2 Criteria for design

The basic requirements that must be met in the foundations for dynamic machines:

Static loads

1. It should be safe against shear failure
2. Basement settlement within normal limits

These requirements are like for usual foundation under static load.

### *Dynamic loads*

1) There should be no resonance; that is, the natural frequency of the machine foundation-soil system should not coincide with the operating frequency of the machine. In fact, a zone of resonance is generally defined and the natural frequency of the system must lie outside this zone. The foundation is high tuned when its fundamental frequency is greater than the operating speed or low tuned when its fundamental frequency is lower than the operating speed. This concept of a high or low tuned foundation is illustrated in Figure 2.

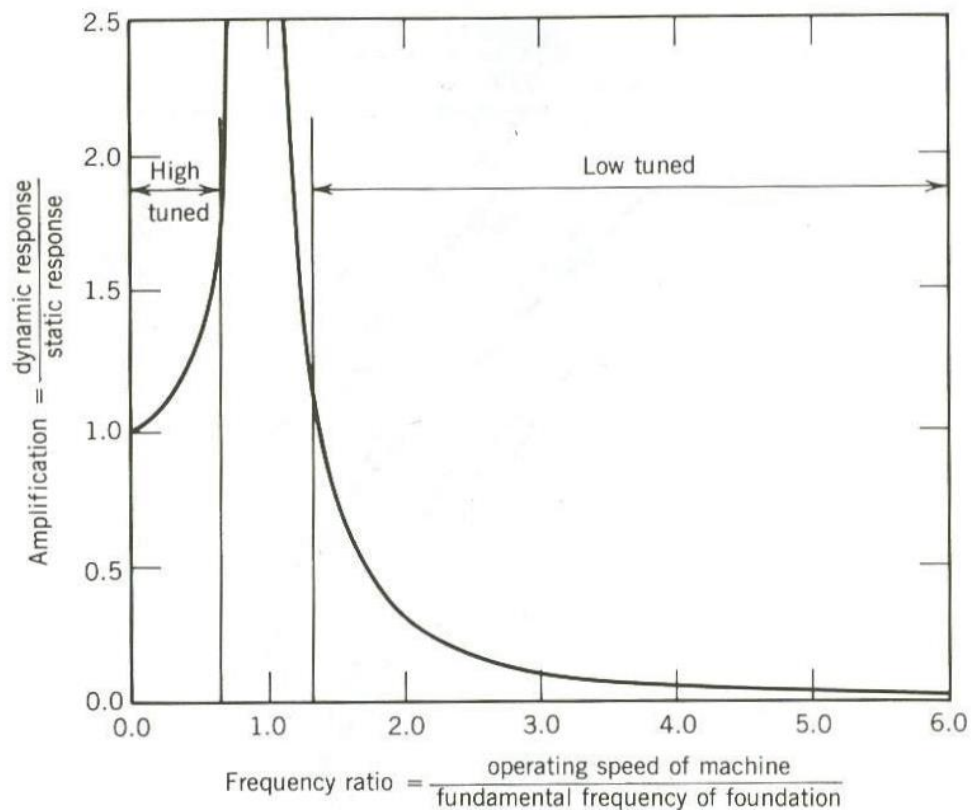


Figure 2. Concept of high or low tuned foundations [3]

2) The amplitudes of motion at operating frequencies should not exceed the limiting amplitudes, which are generally specified by machine manufacturers. If the computed amplitude is within tolerable limits, but the computed natural frequency is close to the operating frequency, it is important that this situation be avoided.

3) The natural frequency of the foundation –soil system should not be whole number multiple of the operating frequency of the machine to avoid resonance with the higher harmonics.



- 4) The settlement should be within permissible limits.
- 5) The combined centre of gravity of machine and foundation should be, to the extent possible, in the same vertical line as the centre of gravity of the base line.
- 6) All rotating and reciprocating parts of the machine should be so balanced that the unbalanced forces and moments are minimised. (This is the responsibility of the mechanical engineers).
- 7) The foundation should be so planned as to permit subsequent alteration of natural frequency by changing the base area or mass of the foundation, if found necessary subsequently. From the practical point of view, the following additional requirements should also be

fulfilled:

- 1) The ground water table should be below the base plane by at least one-fourth of the width of the foundation. Since ground-water is a good conductor of waves, this limits the propagation of vibration.
- 2) Machine foundations should be separated from adjacent building components by means of expansion joints.
- 3) Any pipes carrying hot fluids, if embedded in the foundation, must be properly isolated.
- 4) The foundation should be protected from machine oil by means of suitable chemical treatment, which is acid-resistant.
- 5) Machine foundations should be taken to a level lower than the level of foundations of adjoining structures. [3]

## **2.1 Design Approach for Machine Foundation**

The task for designing of foundations for machines with dynamic loads in addition to the general data on the equipment should contain:

- 1) the technical characteristics of machines (name, type, number of rotations per minute, power, total weight, etc.)
- 2) information about the magnitudes, places of application and directions of action of static loads, as well as about amplitudes and frequencies, places of application and direction of dynamic loads.
- 3) data on the maximum permissible deformations of the foundations and their basement.
- 4) requirements for placement of machine on the foundations.

- 5) drawings of the dimensions of the foundation within the location of the machine, fasteners, as well as auxiliary equipment, communications and hole sizes.
- 6) drawings of all communications adjacent to the foundation.
- 7) data on the geological conditions of the construction site and soil properties.
- 8) information about the location of the designed foundation relative to the nearest structures.
- 9) requirements for the protection of the foundation from groundwater, as well as other external factors characteristic of the design environment.

### **3 General theory**

The analysis of machine foundation is usually performed by idealizing it as a simple system as explained here. Figure 5 shows a schematic sketch of a rigid concrete block resting on the ground surface and supporting a machine. Let us assume that the operation of the machine produces a vertical unbalanced force which passes through the combined centre of gravity of the machine-foundation system. Under this condition, the foundation will vibrate only in the vertical direction about its mean position of static equilibrium. The vibration of the foundation results in transmission of waves through the soil. These waves carry energy with them. This loss of energy is termed „geometrical damping“. The soil below the footing experiences cyclic deformations and absorbs some energy which is termed „material damping“. The material damping is generally small compared to the geometrical damping and may be neglected in most cases. However, material damping may also become important in some cases of machine foundation vibrations. The problem of a rigid block foundation resting on the ground surface, (Figure 3a) may therefore be represented in a reasonable manner by a spring-mass-dashpot system shown in Figure 3b. The spring in this figure is the equivalent soil spring which represents the elastic resistance of the soil below the base of the foundation. The dashpot represents the energy loss or the damping effect. The mass in Figure 3b is the mass of the foundation block and the machine. If damping is neglected, a spring-mass system shown in Figure 3c may be used to represent the problem defined in Figure 3a. Single degree of freedom models shown in Figure 3b and c may in fact be used to represent the problem of machine foundation vibration in any

mode of vibration if appropriate values of equivalent soil spring and damping constants are used. [3]

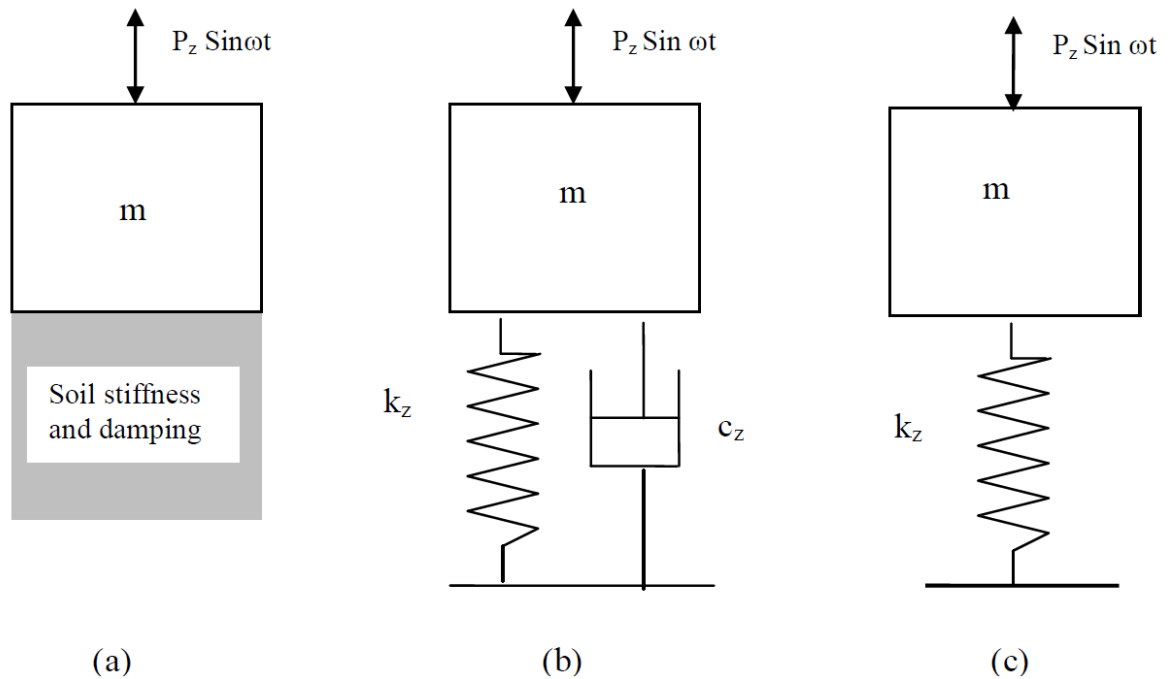


Figure 3. Vertical vibrations of a machine foundation (a) Actual case, (b) Equivalent model with damping, (c) Model without damping [3]

All foundations in practice are placed at a certain depth below the ground surface. As a result of this embedment, the soil resistance to vibration develops not only below the base of the foundation but also along the embedded portion of the sides of the foundation. Similarly the energy loss due to radiation damping will occur not only below the foundation base but also along the sides of the foundation. The type of models shown in Figure 3 b and c may be used to calculate the response of embedded foundations if the equivalent soil spring and damping values are suitably modified by taking into account the behavior of the soil below the base and on the sides of the foundation.

A typical concrete block is regarded as rigid as compared to the soil over which it rests. Therefore, it may be assumed that it undergoes only rigid-body displacements and rotations. Under the action of unbalanced forces, the rigid block may thus undergo displacements and oscillations as follow (Figure 4) [3]

The cyclic displacement of a foundation can have six possible modes:

1. Translation in the vertical direction
2. Translation in the longitudinal direction
3. Translation in the lateral direction
4. Rotation about the vertical axis (yawing)
5. Rotation about the longitudinal axis (rocking)
6. Rotation about the lateral axis (pitching)

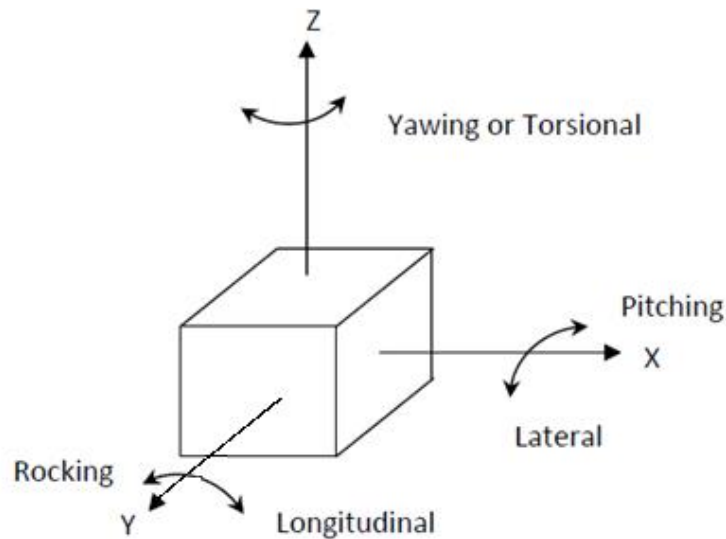


Figure 4. Possible modes for cyclic displacement of foundation [10]

Vertical vibrations:

Figure 3 shows a foundation resting on a soil that can be approximated to be an equivalent spring and dashpot. This foundation is being subjected to a sinusoidally varying force  $P = P_o \sin \omega t$ . The differential equation of motion for this system can be given by

$$m\ddot{z} + c\dot{z} + kz = P_z \sin \omega t$$

Inertial
damping
spring
exciting  
force
force
force
force

where  $\omega$  = circular frequency of vibration (rpm).

can be solved to obtain the amplitude (i.e., maximum displacement) of vibration  $Z$  of the foundation as

$$A_z = \frac{P_z}{k_z \sqrt{\left[1 - \left(\frac{\omega^2}{\omega_{nz}^2}\right)\right]^2 + \left(2\xi_z \frac{\omega}{\omega_{nz}}\right)^2}}$$

Where  $\omega_n = \sqrt{\frac{k}{m}}$  is the undamped natural frequency and  $\xi$  is the damping ratio.

### Torsional vibrations

A block foundation undergoing torsional vibrations is shown in Figure 5. Non-uniform shearing resistance is mobilized during such vibrations.

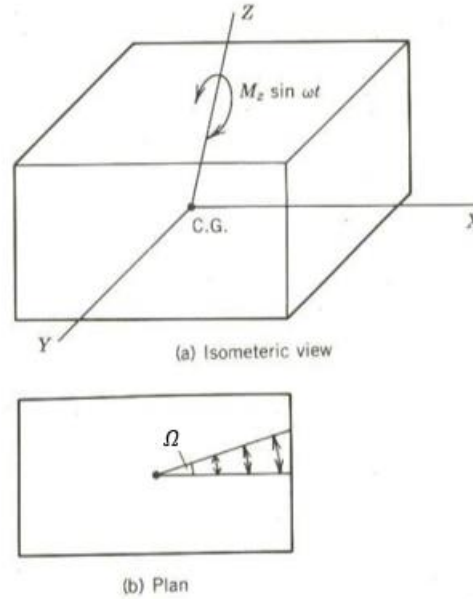


Figure 5. Torsional vibrations of a machine foundation (a) Actual case, (b) Equivalent model [3]

The equation of motion is

$$M_{mz} \ddot{\Omega} + C_{\Omega} \dot{\Omega} + k_{\Omega} \Omega = M_z e^{i\omega t}$$

Damped amplitude of rocking vibrations  $A_{\Omega}$  is given by

$$A_{\Omega} = \frac{M_z}{k_{\Omega} \sqrt{\left[1 - \left(\frac{\omega^2}{\omega_{n\Omega}^2}\right)\right]^2 + \left(2\xi_{\Omega} \frac{\omega}{\omega_{n\Omega}}\right)^2}}$$

Where  $\omega_n = \sqrt{\frac{k}{m}}$  is the undamped natural frequency and  $\xi$  is the damping ratio.

Rocking Vibrations:

A rigid block foundation undergoing rocking vibrations due to an exciting moment  $M_y \sin \omega t$  is shown in Figure 6.

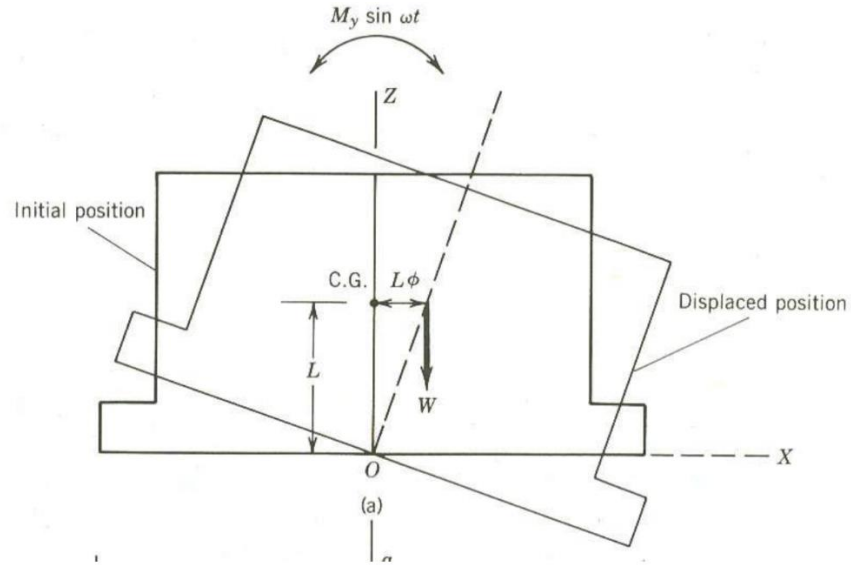


Figure 6. Rocking vibrations of a rigid block under excitation due to an applied moment [3]

The differential equation of motion for this system can be given by

$$M_{mo} \ddot{\phi} + c_{\phi} \dot{\phi} + k_{\phi} \phi = M_y \sin \omega t$$

Damped amplitude of rocking vibrations  $A_{\phi}$  is given by

$$A_{\phi} = \frac{M_y}{k_{\phi} \sqrt{\left(1 - \left(\frac{\omega}{\omega_{n\phi}}\right)^2\right)^2 + \left(2\xi_{\phi} \frac{\omega}{\omega_{n\phi}}\right)^2}}$$

Where  $\omega_n = \sqrt{\frac{k}{m}}$  is the undamped natural frequency and  $\xi$  is the damping ratio

### Horizontal translation:

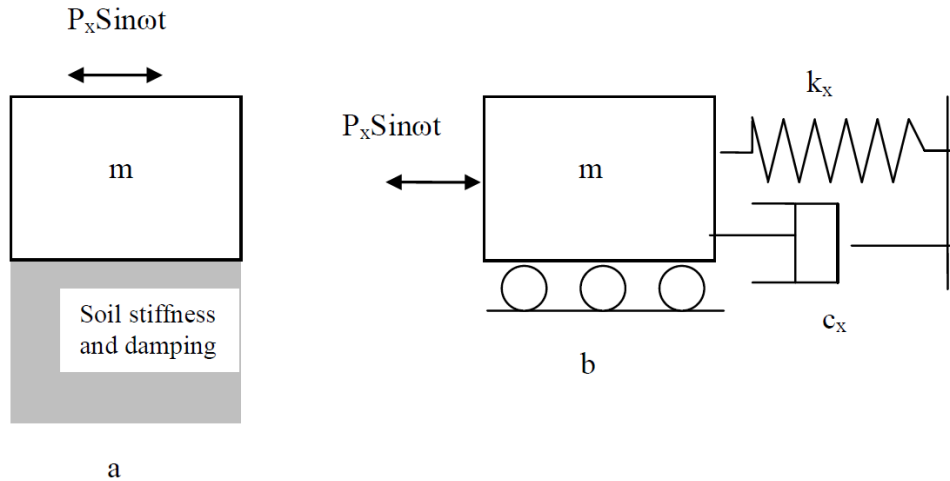


Figure 7. Sliding vibrations of a machine foundation (a) Actual case, (b) Equivalent model [3]

The equation of the analog for sliding is

$$m\ddot{x} + c_x \dot{x} + k_x x = P_x \sin \omega t$$

The damped amplitude in sliding is obtained as:

$$A_x = \frac{P_x}{k_x \sqrt{\left(1 - \left(\frac{\omega}{\omega_{nx}}\right)^2\right)^2 + \left(2\xi_x \frac{\omega}{\omega_{nx}}\right)^2}}$$

Where  $\omega_n = \sqrt{\frac{k}{m}}$  is the undamped natural frequency and  $\xi$  is the damping ratio.

## 4 Design parameters

### 4.1 Dynamic soil properties

Soil dynamics deals with engineering properties and behavior of soil under dynamic stress. For the dynamic analysis of machine foundations, soil properties, such as Poisson's ratio, dynamic shear modulus, soil density and damping of soil, are generally required. Satisfactory design of a machine foundation needs information on soil profile, depth of different layers, physical properties of soil and ground water level. This information can be obtained by usual sub-surface exploration techniques. In addition, one must determine dynamic shear modulus, material damping, poisons ratio and mass density of soil for dynamic analysis of the machine foundation. Dynamic shear modulus of

a soil is generally determined from laboratory or field tests. Material damping can be determined from vibration tests on soil columns in the laboratory. The values of dynamic shear modulus and damping may be estimated from empirical estimations for preliminary design purposes. [4]

The soil characteristics needed in analysis of foundation are:

1. Shear modulus  $G$ , ( $\text{kN/m}^2$ )

2. Poisson's ratio  $\mu$

Typical values for Poisson's ratio:

- Soft clay: 0.3-0.4
- Medium dens sand/gravel: 0.3-0.4
- Dense sand or gravel: 0.4-0.5
- Partially saturated clay: 0.35-0.45
- Saturated clay: 0.4-0.5

3. Damping of soil  $\xi$

4. Soil density  $\rho$  ( $\text{kg/m}^3$ )

Typical values for soil mass density:

- Concrete or solid rock clay:  $2400 \text{ kg/m}^3$
- Clay:  $1000-1100 \text{ kg/m}^3$
- Packed sand or gravel:  $11600- 1900\text{kg/m}^3$

5. Allowable soil pressure  $S_{\text{all}}$ , ( $\text{kN/m}^2$ )

## 4.2 Machine parameters

The machine parameters needed in analysis of foundation are:

- Static weight of machine and accessories.
- Characteristics of dynamic loads imposed by the machine operation and their point of application.
- Rotation speed of machine.

Generally, the weight of the machine, center of gravity, surface areas, and operating speeds are readily available from the manufacturer of the machine.



## 5 The preliminary design

- For preliminary design of concrete block foundations, the length and width shall be 300- 600 mm longer and wider, respectively, than the equipment (if block mounted) or the skid (if skid mounted).
- Block foundations shall have a minimum of 50% of the block thickness embedded in the soil, unless otherwise specified by the equipment user.
- Depth shall be a minimum as follows:

1.2-1.5 m for drivers less than 1840 kW;

1.8 m for drivers in the range of 1840-3680 kW

1.8-2.5 m for drivers greater than 3680 kW

- The width of the foundation shall be at least 1.5 times the vertical distance from the base to the machine center line, see Figure 8, unless analysis demonstrated that a lesser value will perform adequately.
- The ratio of the height of the machine crankshaft above the base to the width of the foundation block (or the pile group in the plan, if applicable) shall not be more than 0.65.
- The top of the finished foundation shall be minimum 100 mm above the finished elevation of the floor slab or grade to prevent damage to the machinery from runoff or wash-down water

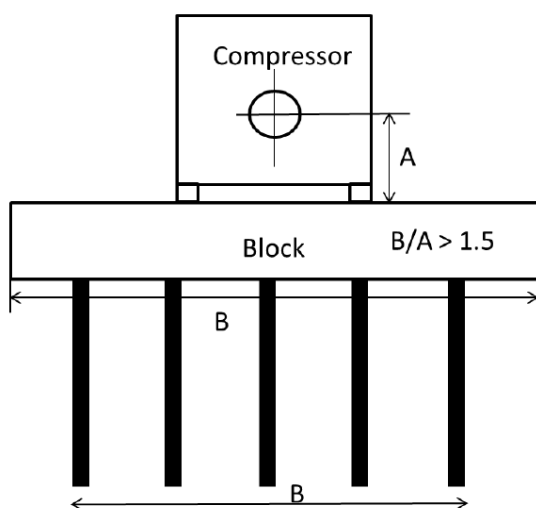


Figure 8. [6]

- To minimize torsional effects, a vertical line drawn through the centroid of the machine or resultant of several machines shall pass within a distance of 5% of the plan dimensions of the base from the centroid of the contact area (or pile group), see Figure 9. [6]

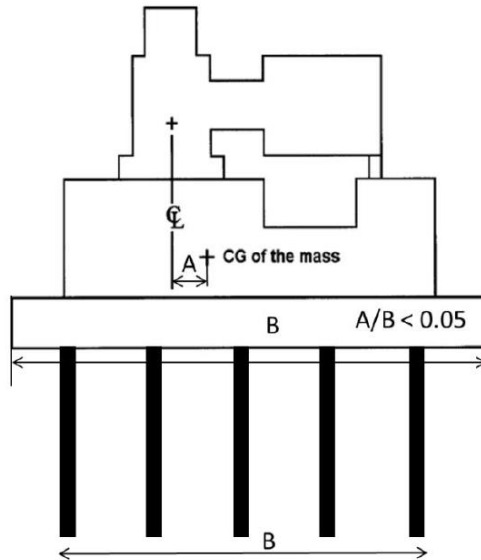


Figure 9. [6]

## 6 Algorithm of calculations

### 6.1 Rotary machines

The dynamic forces acting at the axis of the shaft of rotary machine will excite the structure in three different modes: vertical, horizontal and rocking. As an example of calculations would be the rectangular foundation for Compressor TYPE 4HF/3 (Figure 10). It is a rotary type machine. All calculations are presented in Excel file.

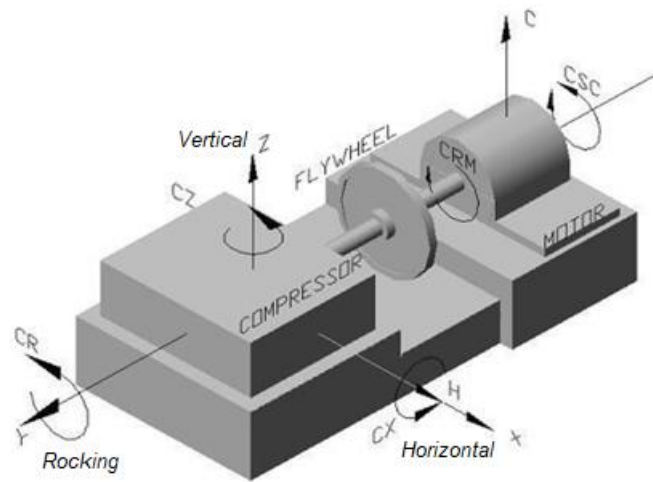


Figure 10. Acting axis for Compressor TYPE 4HF/3

Design data:

Static weight of machinery: 196,13 kN (20 tons)

Rotation speed of machinery: 327rpm

Vibrating forces from machine parameters: Vertical  $F_{z0}=2,793$  kN, Horizontal  $F_{x0}=62,488$  kN

Density of concrete:  $24,516$  kN/m<sup>3</sup> (2500 kg/m<sup>3</sup>)

Soil density  $\rho$ : 18 kN/m<sup>3</sup>

Shear Modulus G: 21000 kN/m<sup>2</sup>

Poisson's ratio  $\mu$ : 0,45

Soil damping: 0,05

Calculations:

Weight of foundation  $W_f$ , (kN)

$$W_f = V_f \cdot \text{density of concrete (kN/m}^3)$$

$W$  = weight of machine+ weight of footing

1. Mass  $m$ , (kN·c<sup>2</sup>/m)

For vertical and horizontal translation:

$$m = \frac{W}{g}, \text{ where } g=9,81 \text{ m/c}^2$$

Mass moment of Inertia for footing and machine

Rocking:  $I_\psi$  According to Appendix 1 (footing)

$$I_\psi = m_{(\text{machine})} * k_i^2 \text{ (machine)}$$

$k_i$  – distance from center of mass to base of footing

1) Equivalent radius  $r_0$ , (m)

Table 1. Determination of equivalent radius

Mode of vibration	Equivalent radius
Vertical	$r_0 = \sqrt{\frac{BL}{\pi}}$
Horizontal	$r_0 = \sqrt{\frac{BL}{\pi}}$
Rocking/Pitching	$r_{\theta,\psi} = \sqrt[4]{\frac{BL^3}{3\pi}}$
Twisting	$r_\Omega = \sqrt[4]{\frac{BL(B^2 + L^2)}{6\pi}}$

2) Mass ratio  $B_z$ ,  $B_x$ ,  $B_\theta$ ,  $B_\psi$

Table 2. Determination of mass ratio

Mode of vibration	Mass ratio
Vertical	$B_z = \frac{(1 - \mu)}{4} \frac{W}{\rho r_0^3}$
Horizontal	$B_x = \frac{7 - 8\mu}{32(1 - \mu)} \frac{W}{\rho r_0^3}$
Rocking/Pitching	$B_{\theta,\psi} = \frac{3(1 - \mu)}{8} \frac{I_\theta}{\rho r_{\theta,\psi}^5}$
Twisting	$B_\Omega = \frac{I_\Omega}{\rho r_0^5}$

**Spring constant calculation:**

- 3) Spring constant coefficient  $\alpha_z$ ,  $\alpha_x$ ,  $\alpha_\theta$ ,  $\alpha_\psi$  from Figure 7 depending on  $L/B$

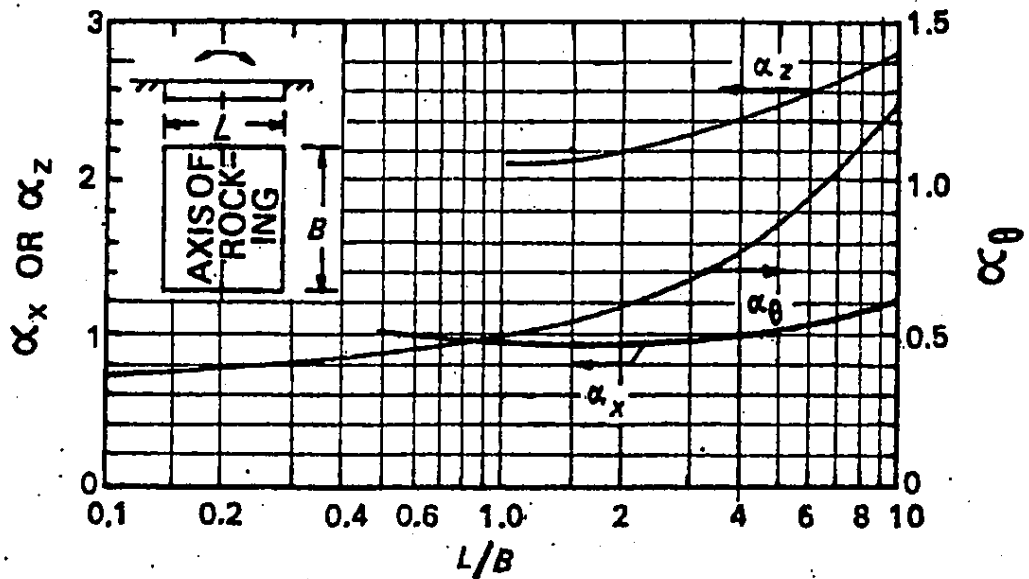


Figure 11. Spring constant coefficient determination [1]

- 4) Spring constant  $k_z$ ,  $k_x$ ,  $k_\theta$ ,  $k_\psi$

Table 3. Determination of spring constant

Mode of vibration	Spring constant
Vertical	$k_z = \frac{G}{1-\mu} \alpha_z \sqrt{BL}$
Horizontal	$k_x = 2(1+\mu)G\alpha_x \sqrt{BL}$
Rocking/Pitching	$k_{\theta,\psi} = \frac{G}{1-\mu} \alpha_{\theta,\psi} BL^2$
Twisting	-

**Damping calculation:**

- 5) Damping  $\xi_z$ ,  $\xi_x$ ,  $\xi_\theta$ ,  $\xi_\psi$

Table 4. Determination of damping

Mode of vibration	Damping
Vertical	$\xi_z = \frac{0,425}{\sqrt{B_z}}$
Horizontal	$\xi_x = \frac{0,288}{\sqrt{B_x}}$
Rocking/Pitching	$\xi_{\theta,\psi} = \frac{0,15}{(1+B_{\theta,\psi})\sqrt{B_{\theta,\psi}}}$

Twisting	$\xi_{\Omega} = \frac{0,5}{1 + 2B_{\Omega}}$
----------	--

6) Total damping

$\xi_{\psi} + \xi$ (from soil data)

7) Natural frequency  $f_0$ , (Hz)

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Where k- Spring constant,(Kn/m) m- mass, (kN·c<sup>2</sup>/m)

8) Resonance Frequency  $f_{res}$ , (Hz)

For vertical translation check:

$2\xi^2 > 1$  means that resonance not possible

$2\xi^2 < 1$  a resonance occurs, increase trial dimensions of foundation or choose a rotation speed of machine so that there will be no resonance

For rocking and horizontal:

$$f_{res} = \frac{f_0}{\sqrt{1 - 2\xi^2}}$$

9) Vibrating forces from machine parameters:

Vertical  $F_{z0}=2,793$  kN

Horizontal  $F_{x0}=62,488$  kN

Rocking  $F_{\psi 0} = F_{x0} \cdot R_v$

$R_v$ - Vertical distance from base to horizontal machine load, m

10) Magnification factor

$$M = \frac{1}{\sqrt{(1-r^2)+(2\xi r)^2}}$$

Where  $r = \frac{\omega}{\omega_n}$ ;  $\omega_n = \sqrt{\frac{k}{m}}$

k- Spring constant,(Kn/m); m- mass, (kN·c<sup>2</sup>/m);  $\omega_n$ -undamped natural circular frequency.

11) Vibration Amplitude, (m or mm, rad)

$$Z = \frac{\sum M_z F_{z0}}{k_z}$$

$$X = \frac{\sum M_x F_{x0}}{k_x}$$

$$\psi = \frac{\sum M_\psi F_{\psi0}}{k_\psi}$$

12) Component of rocking oscillation

At edge of footing=  $\psi \cdot R_h$

At center of bearing=  $\psi \cdot R_v$

$R_h$ - Horizontal distance from center to edge of footing in the x-direction, m

$R_v$ - Vertical distance from base to horizontal machine load, m

13) Resultant vibration amplitude

$Z_t = Z +$  (At edge of footing=  $\psi \cdot R_h$ )

$X_t = X +$  (At center of bearing=  $\psi \cdot R_v$ )

14) Transmissibility Factor

$$T_{z,x,\psi} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

15) Transmitted force

$$P_V = \sum T F_{z0}, kN$$

$$P_H = \sum T F_{x0}, kN$$

$$P_T = \sum T F_{\psi0}, kN * m$$

Check for:

1) Static conditions

✓ Static bearing capacity. Proportion of footing area for 50% of  $S_{all}$

$$P_{stat} < 0,5 * S_{all}$$

Where  $P_{stat} = W(\text{footing} + \text{machine}) / (L * B)$

✓ Static settlement must be uniform; C.G. of footing and machine should be within 5% of each linear dimension.

✓ Bearing capacity: static plus dynamic loads. The Magnification factor should preferably be less than 1,5. The sum of static and dynamic loads should

be within of the footing C.G. For 75% of the  $S_{all}$  given for the static load

$$\text{condition} = P_{stat} + \frac{P_V}{L*B} \pm \frac{P_T*0,15}{L*B^2} < 0,75*S_{all}$$

- ✓  $W_{footing}/W_{machine}$  should be  $\geq 3$
- ✓  $P_{stat} - \frac{P_V}{L*B} - \frac{P_T*0,15}{L*B^2} > 0$ , means that there should be no uplift.
- 2) Limiting dynamic conditions
  - ✓ Resonance. The acting frequencies of the machine should have at least difference of  $\pm 20\%$  with the resonance frequency.  $0,8f_{res} \geq f \geq 1,2f_{res}$  [1]

## 6.2 Reciprocating machines

Forces generated by the reciprocating compressor will result in vertical, lateral, rocking and pitching oscillation of the footing, while the forces generated by the motor only will tend to oscillate the footing in the vertical, longitudinal, torsional and pitching modes. However, the magnitude of the forces generated by the motor are small and, consequently, the resulting vibration response is negligible. Therefore, a dynamic analysis is only performed for the compressor forces. [1]

In this paragraph only differences between calculation algorithm of Reciprocating and Rotary machines will be described. All other calculations are the same as described above.

Spring constant calculation

- 1) Embedment coefficients for spring constants  $\eta_z, \eta_x, \eta_\theta, \eta_\psi$

Table 5. Determination of embedment coefficients for spring constants

Mode of vibration	Embedment coefficients
Vertical	$\eta_z = 1 + 0,6(1-\mu)(h/r_0)$
Horizontal	$\eta_x = 1 + 0,55(2-\mu)(h/r_0)$
Rocking/Pitching	$\eta_{\theta,\psi} = 1 + 1,2(1-\mu)(h/r_{e,\psi}) + 0,2(2-\mu)(h/r_{e,\psi}^2)$
Twisting	-



2) Spring constant  $k_z, k_x, k_\theta, k_\psi$

Table 6. Determination of spring constant

Mode of vibration	Spring constant
Vertical	$k_z = \frac{G}{1-\mu} \alpha_z \sqrt{BL} \eta_z$
Horizontal	$k_x = 2(1+\mu) G \alpha_x \sqrt{BL} \eta_x$
Rocking/Pitching	$k_\theta = \frac{G}{1-\mu} \alpha_{\theta,\psi} B L^2 \eta_{\theta,\psi}$
Twisting	-

Damping calculation

3) Damping ratio embedment factor  $\alpha_z, \alpha_x, \alpha_\theta$

Table 7. Determination of embedment factor

Mode of vibration	Damping ratio embedment factor
Vertical	$\alpha_z = \frac{1 + 1,9(1-\mu)\left(\frac{h}{r_0}\right)}{\sqrt{\eta_z}}$
Horizontal	$\alpha_x = \frac{1 + 1,9(2-\mu)\left(\frac{h}{r_0}\right)}{\sqrt{\eta_x}}$
Rocking/Pitching	$\alpha_{\theta,\psi} = \frac{1 + 0,7(1-\mu)\left(\frac{h}{r_0}\right) + 0,6(2-\mu)\left(\frac{h}{r_0}\right)^3}{\sqrt{\eta_{\theta,\psi}}}$

4) Effective damping coefficients  $n_\psi, \theta$

Depending on Mass ratio B

Values of  $n_\psi$  for Various Values of  $\beta_\psi^*$

$B_\psi$	5	3	2	1	0.8	0.5	0.2
$n_\psi$	1.079	1.110	1.143	1.219	1.261	1.378	1.600

Figure 12. [1]

5) Damping  $\xi_z, \xi_x, \xi_\theta, \xi_\psi$

Table 8. Determination of damping

Mode of vibration	Damping
Vertical	$\xi_z = \frac{0,425}{\sqrt{B_z}} \alpha_z$
Horizontal	$\xi_x = \frac{0,288}{\sqrt{B_x}} \alpha_x$

Rocking/Pitching	$\xi_{\theta,\psi} = \frac{0,15\alpha_{\theta}}{(1 + n_{\theta,\psi}B_{\theta,\psi})\sqrt{n_{\theta,\psi}B_{\theta,\psi}}}$
Twisting	$\xi_{\Omega} = \frac{0,5}{1 + 2B_{\Omega}}$

6) Resonance Frequency  $f_{res}$ , (Hz)

For vertical and horizontal translation check:

$2\xi^2 > 1$  If yes, resonance not possible

$2\xi^2 < 1$  a resonance occurs, increase trial dimensions of foundation or choose a rotation speed of machine so that there will be no resonance

For rocking and pitching:

$$f_{res} = \frac{f_0}{\sqrt{1 - 2\xi^2}}$$

7) Displacement Response, (m or mm, rad)

$$Z = \frac{\sum M_z F_{z0}}{k_z}$$

$$X = \frac{\sum M_x F_{x0}}{k_x}$$

$$\theta = \frac{\sum M_{\theta} F_{\theta 0}}{k_{\theta}}$$

$$\psi = \frac{\sum M_{\psi} F_{\psi 0}}{k_{\psi}}$$

8) Total displacement, (m or mm)

$$Z_t = Z + \theta \cdot R_{hx} + \theta \cdot R_{hy}$$

$$X_t = X + \psi \cdot R_v$$

$$Y_t = \theta \cdot R_v$$

$R_{hy}$ - Horizontal distance from center to edge of footing in the y-direction, m

$R_{hx}$ - Horizontal distance from center to edge of footing in the x-direction, m

R<sub>v</sub>- Vertical distance from base to horizontal machine load, m

9) Transmissibility Factor

$$T_{z,x,\theta,\psi} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

10) Transmitted force

$$P_V = \sum T F_{z0}, kN$$

$$P_R = \sum T F_{x0}, kN$$

$$P_{T\psi} = \sum T F_{z\psi}, kN * m$$

$$P_{T\theta} = \sum T F_{z\theta}, kN * m$$

Check for:

1) Static conditions

✓ Static bearing capacity. Proportion of footing area for 50% of S<sub>all</sub>

$$P_{stat} < 0,5 * S_{all}$$

Where  $P_{stat} = W(\text{footing} + \text{machine}) / (L * B)$

✓ Static settlement must be uniform; C.G. of footing and machine should be within 5% of each linear dimension

✓ Bearing capacity: static plus dynamic loads. The Magnification factor should preferably be less than 1,5. The sum of static and dynamic loads should not create bearing pressure greater than 75% of the S<sub>all</sub>.

Transmitted Dynamic Vertical force P<sub>v</sub>

Transmitted moments P<sub>Tψ2</sub> = P<sub>Tψ</sub> + P<sub>R</sub>\* R<sub>v</sub> + P<sub>v</sub>\*(dimension from CG of compressor to combination CG of compressor and motor)

$$P_{dyn} = \frac{P_V}{L * B} \pm \frac{P_{T\theta} * 0,15}{L * B^2} \pm \frac{P_{T\psi2} * 0,15}{L^2 * B}$$

Total static plus dynamic bearing pressure = P<sub>dyn1</sub> + P<sub>stat</sub> should be < 0,75 \* S<sub>all</sub>

✓ Total upward pressure = P<sub>dyn2</sub> + P<sub>stat</sub> should be > 0, means that there is no uplift

✓ W<sub>footing</sub> / W<sub>machine</sub> should be ≥ 3

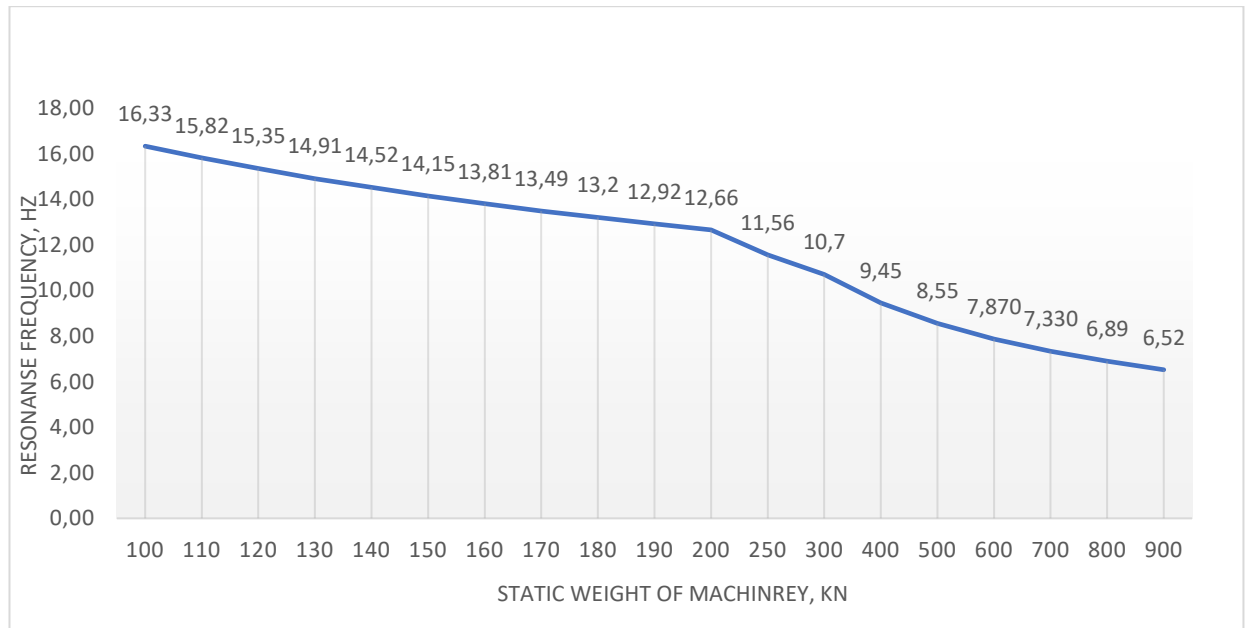
2) Limiting dynamic conditions

✓ Resonance. The acting frequencies of the machine should have at least difference of ±20% with the resonance frequency.  $0,8f_{res} \geq f \geq 1,2f_{res}$  [1]

## 7 Analysis

Design of dynamically loaded foundations is a complex and volume process; all factors bear their meanings. How do machine or soil data affect the calculation and what would it be to change only one of them? Let us take a few characteristics and find out the relations. As it was mentioned above amplitudes and frequencies are one of the most important and decisive parameters.

Graph 1. Influence of machine weight on resonant frequency.



With all constant parameters of foundation and soil, check how does the static weight of machine affect the foundation-machine-soil system resonance check. The rotation speed of the machine is 327 rpm or  $f=5,45$  Hz. Increasing the weight of the machine decreases the resonance frequency so that the frequency of the foundation is close to the frequency of the machine. To avoid resonance, the difference between these two frequencies should be 20% less or more:  $0,8f_{res} \geq f \geq 1,2f_{res}$ . So that when the value of the machine weight is 900kN, the resonant frequency is  $f_{res}=6,52$ Hz. And the resonant check is:  $6,52 \cdot 0,8 = 5,216 < 5,45$ ; resonance is possible. However, this situation is unlikely, because the ratio of the weight of the foundation to the weight of the machine must be greater than 3. This condition provides a well-balanced system. In this case, the weight of the machine is too big. The diagram shows a large margin

relative to the weight of the machine: the weight of the machine must be too large not to pass the resonance check.

## **8 Material recommendations**

### **8.2 Concrete**

It is recommended to use concrete C30/37. The design rules are presented in Eurocode 2: Design of concrete structures SFS-EN1992-1-1

### **8.3 Reinforcement**

It is recommended to use bars a500hw.

### **8.4 Grout**

Grouting is used to completely fill the gap between a non-precise top elevation of a concrete foundation block and the machined bottom of the equipment base. Grouting materials (either cement or epoxy based) have to withstand vertical load, while horizontal movement of a machine is restrained by the clamping force of the properly torqued anchoring system against the machinery grout. Grouting material must be able to withstand prolonged compressive loads, at equipment-operating temperatures, without creeping or allowing the base to deflect, as this will disturb the alignment of the moving parts of the machine. Grouting also provides an inert, impervious barrier between machine fluids and concrete block.

It is recommended to use MasterFlow648. MasterFlow 648 is an epoxy resin-based precision grout used to secure critical equipment for proper alignment and transmission of static and dynamic loads. With carefully balanced physical properties and excellent resistance to chemical attack, elevated service temperatures, vibration and torque, MasterFlow 648 is formulated for easy installation, with good flow characteristics suitable for pouring or pumping, low dust generation and soap and water cleanup. [8]

## **9 Pile foundations**

*Pile foundations (Figure 13)*

Any of the foundation types mentioned above can be supported directly on the ground or on piles. It is advisable to build a pile foundation in cases where an

incompressible soil layer is so deep that it is impossible to build other types of foundations, namely in the case of building a foundation on weak soils (for example, on peat soils or in swampy areas). Piles transfer stresses to the base soils through the side surface (by friction) and through the lower end. Various types of piles are used including drilled piers, auger cast piles, and driven piles. Piling is required when the soil is:

- too soft to support the combined deadweight of machinery and foundation so that the soil bearing capacity or the limits for settlement are exceeded;
- the soil is not stiff enough to resist vibration forces and deflections;
- water table is too high or variable to assure consistent soil properties over time.

The introduction of piles in a soil stratum makes the soil system stiffer. Due to this both natural frequency and amplitude of vibration are affected. In all vibration related problems, resonance needs to be avoided. Therefore, the natural frequency of the soil-pile-foundation system has to be evaluated in analysis and design. Dynamic analysis of a soil-pile-foundation system requires an evaluation of the natural frequency and displacement amplitudes of the system for a particular loading sequence. [9]

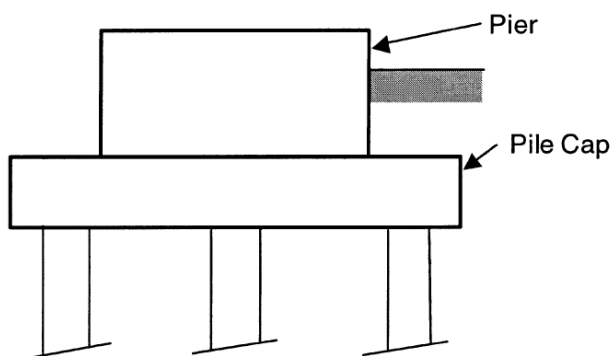


Figure 13. Pile foundation [4]

Example of calculation spring and damping constants for the pile-supported block of Figure 14.

Given data:

Piles: precast concrete 300x300 mm square

$L_p=9,1$  m

$E_p=27800$  MPa  $\gamma_p=23,6$  kN/m<sup>3</sup>  $\mu_p=0,15$

Soil:  $G'=17700$  kPa  $\mu_s=0,33$   $\gamma_s=16,5$  Kn/m<sup>3</sup>

Other:  $\omega=179,2$  rad/sec

Solution:

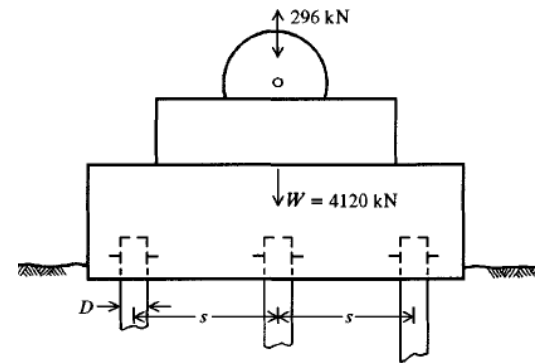
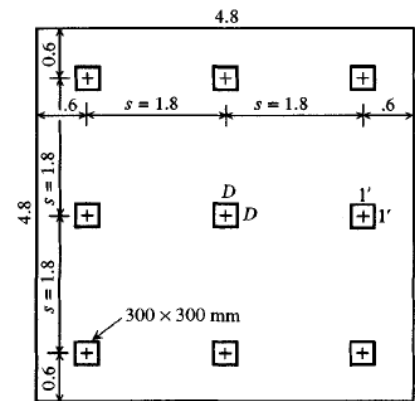


Figure 14. [5]

$$r_0 = \sqrt{\frac{A_p}{\pi}} = \sqrt{\frac{0,3 * 0,3}{\pi}} = 0,169m$$

$$\frac{L_p}{r_0} = \frac{9,1}{0,169} = 54 > 30 \text{ (OK to use Figure 15)}$$

$$V_s = \sqrt{\frac{G'}{\gamma_s}} = \sqrt{\frac{17700 * 9,81}{16,5}} = 103 \text{ m/sec}$$

$$V_p = \sqrt{\frac{E_p}{\gamma_p}} = \sqrt{\frac{27,8E + 6 * 9,81}{23,6}} = 3400 \text{ m/sec}$$

$$\frac{V_s}{V_p} = \frac{103}{3400} = 0,0303 \text{ (use 0,030 for Figure 15)}$$

$$\frac{\gamma_s}{\gamma_p} = \frac{16,5}{23,6} = 0,70$$

$$I_p = \frac{bh^3}{12} = \frac{0,3^4}{12} = 0,6750E - 3 \text{ m}^4$$

For torsion constant J use an equivalent round pile based on  $r_0$ , or

$$J = \frac{\pi r_0^4}{2} = \frac{\pi * 0,169^4}{2} = 0,1281E - 2 \text{ m}^4$$

The dimensionless frequency factor  $a_0$  is computed as

$$a_0 = \frac{\omega r_0}{V_s} = \frac{179,2 * 0,169}{103} = 0,29$$

With this several data computed, we can compute the several spring and damping constants using equations given in Figure 15 with  $f_{i,j}$  values from Figure 16 (E,A,I,J are pile values):

Vertical:

$$K_z = \frac{EA}{r_0} f_{18,1} = \frac{27800 * 0,3^2}{0,169} * 0,037 = 547,8 \text{ MN/m}$$

$$c_z = \frac{EA}{V_s} f_{18,2} = \frac{27800 * 0,09}{103} * 0,016 = 0,389 \text{ MN * s/m}$$

Horizontal:

$$K_h = \frac{EI}{r_0^3} f_{11,1} = \frac{17800 * 0,6750E - 3}{0,169^3} * 0,018 = 44,8 \text{ MN/m}$$

$$c_h = \frac{EI}{r_0^2 V_s} f_{11,2} = \frac{12,015}{0,169^2 * 103} * 0,043 = 0,1756 \text{ MN * s/m}$$

Rocking:

$$K_\theta = \frac{EI}{r_0} f_{7,1} = \frac{12,015}{0,169} * 0,345 = 24,53 \text{ MN * m}$$

$$c_\theta = \frac{EI}{V_s} f_{7,2} = \frac{12,015}{103} * 0,245 = 0,02858 \text{ MN * s * m}$$

Torsion:

$$G'_p = \frac{E_p}{2(1 + \mu_p)} = \frac{27800}{2 * 1,15} = 12000 \text{ MPa}$$

$$K_\Omega = \frac{G'_p J}{r_0} f_{12,1} = \frac{12000 * 0,1281E - 2}{0,169} * 0,105 = 9,55 \text{ MN * m}$$

$$c_\Omega = \frac{G'_p J}{V_s} f_{12,2} = \frac{15,372}{103} * 0,011 = 0,00164 \text{ MN * s * m}$$

With these data and the large s/D ratio = 1,8/0,3=6, the vertical spring and damping constants will be summed to obtain a global value for the nine piles as

$$K_z = 9 * 547,8 = 4930,2 \text{ MN/m}$$

$$c_z = 9 * 0,389 = 3,501 \text{ MN*s/m}$$

We can compute the block mass from the weight of block and machinery shown in Figure 16.

$$m = 4,12/9,81 = 0,4201 \text{ MN*s}^2/\text{m}$$

and, using formula

$$z = \frac{F_o}{\sqrt{(K_z - m\omega^2)^2 + c_z^2 \omega^2}}$$

$$z = \frac{0,296}{\sqrt{[4930,2 - (0,4201 * 179,2)^2]^2 + (3,501 * 179,2)^2}} = 0,3058E - 3 \text{ m or } 0,306 \text{ mm}$$



Mode	Spring $K_i$	Damping $c_i$
Vertical	$K_z = \frac{EA}{r_o} f_{18,1}$	$c_z = \frac{EA}{V_s} f_{18,2}$
Horizontal	$K_h = \frac{EI}{r_o^3} f_{11,1}$	$c_h = \frac{EI}{r_o^2 V_s} f_{11,2}$
Rocking	$K_\theta = \frac{EI}{r_o} f_{7,1}$	$c_\theta = \frac{EI}{V_s} f_{7,2}$
Cross-stiffness/damping	$K_{x\theta} = \frac{EI}{r_o^2} f_{9,1}$	$c_{x\theta} = \frac{EI}{r_o V_s} f_{9,2}$
	$K_{\theta x} = K_{x\theta}$	$c_{\theta x} = c_{x\theta}$
Torsion	$K_t = \frac{G'J}{r_o} f_{12,1}$	$c_t = \frac{G'J}{V_s} f_{12,2}$

Figure 15. Pile spring and damping constants [9]

$V_s/V_c$	Stiffness					Damping				
	$f_{18,1}$	$f_{7,1}$	$f_{9,1}$	$f_{11,1}$	$f_{12,1}$	$f_{18,2}$	$f_{7,2}$	$f_{9,2}$	$f_{11,2}$	$f_{12,2}$
0.01	0.034	0.199	-0.019	0.004	0.045	0.002	0.136	-0.028	0.008	0.002
0.02	0.035	0.282	-0.038	0.010	0.072	0.007	0.198	-0.056	0.023	0.007
0.03	0.037	0.345	-0.057	0.018	0.105	0.016	0.245	-0.084	0.043	0.011
0.04	0.040	0.398	-0.076	0.027	0.139	0.027	0.283	-0.112	0.066	0.015
0.05	0.044	0.445	-0.095	0.038	0.174	0.041	0.314	-0.141	0.092	0.019
0.06	0.049	0.448	-0.114	0.050	0.208	0.055	0.346	-0.169	0.122	0.022

Figure 16.  $f_{i,j}$  values [9]

## 10 Conclusion

The foundations of machines with dynamic loads are sources of waves propagating in the soil, which have a detrimental effect on the nearby structures of buildings and structures, objects with equipment and equipment that is sensitive to vibrations, as well as residential buildings.

Vibrations spreading from machine foundations can cause uneven precipitation of foundations and additional stresses in nearby buildings and structures, which cause cracks in them and even their destruction, affecting the operation of

some machines (for example, precise machines), measuring equipment and etc.

To reduce the level of propagating vibrations, various measures are used: choose the most rational dimensions and configurations of the foundation, change the rigidity of the base, connect several foundations with a common slab, apply active and passive vibration isolation, dynamic oscillation dampers and attached plates that balance counterweights, adjust the rotational speed of the machines, etc.

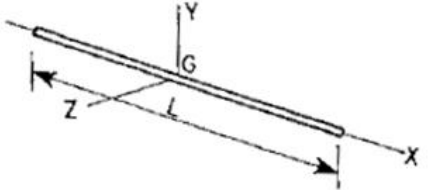
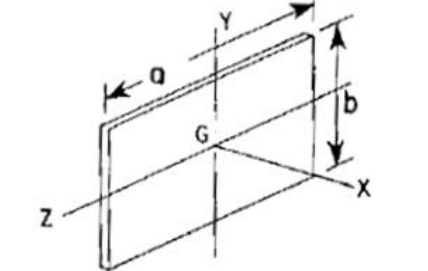
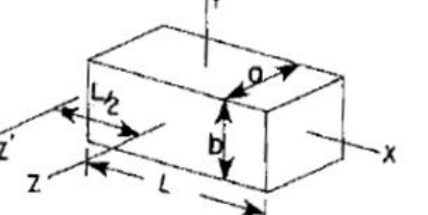
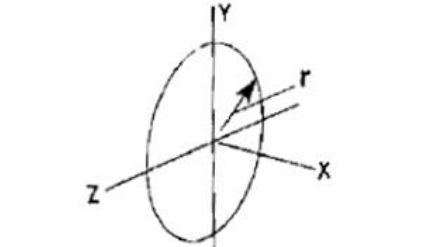

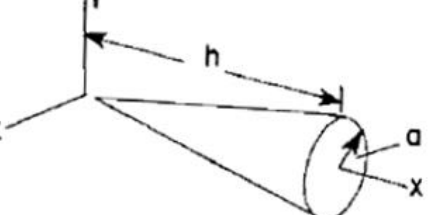
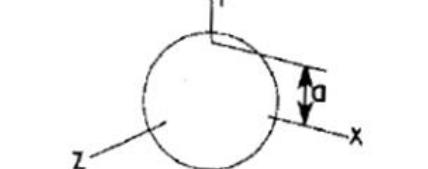
The work presents various design criteria and methods and procedures of analysis, design, and construction currently applied to dynamic equipment foundations by industry practitioners. This document provides general guidelines with reference materials, rather than specifying requirements for adequate design. [4]

## List of references

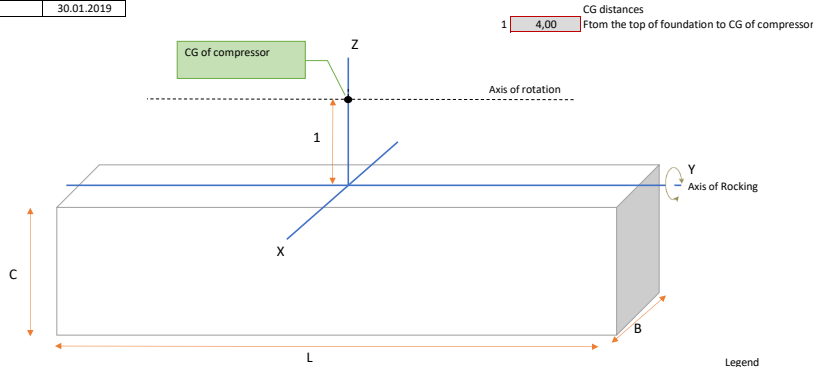
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## Appendices

### Appendix 1 Mass moment of Inertia of common geometric shapes

SLENDER ROD		$I_y = I_z = \frac{1}{12} mL^2$
THIN RECTANGULAR PLATE		$I_x = \frac{1}{12} m (a^2 + b^2)$ $I_y = \frac{1}{12} ma^2$ $I_z = \frac{1}{12} mb^2$
RECTANGULAR PRISM		$I_x = \frac{1}{12} m (a^2 + b^2)$ $I_y = \frac{1}{12} m (a^2 + L^2)$ $I_z = \frac{1}{12} m (b^2 + L^2)$ $I_{z'} = I_z + mL^2/4$
THIN DISK		$I_x = \frac{1}{2} mr^2$ $I_y = I_z = \frac{1}{4} mr^2$
CIRCULAR CYLINDER		$I_x = \frac{1}{2} ma^2$ $I_y = I_z = \frac{1}{12} m (3a^2 + L^2)$ $I_{z'} = I_z + mL^2/4$
CIRCULAR CONE		$I_x = \frac{3}{10} ma^2$ $I_y = I_z = \frac{3}{5} m (\frac{1}{4} a^2 + h^2)$
SPHERE		$I_x = I_y = I_z = \frac{2}{5} ma^2$

Author	Date
Arina Belovolova	30.01.2019



CG distances  
1 4,00 From the top of foundation to CG of compressor

Legend  
value must be entered manually

Density of concrete 24,516 kN/m<sup>3</sup>

Static weight of machinery  
W= 196,13 kN  
Mass of machinery  
m= 19,99 kN-sec<sup>2</sup>/m

Static weight of footing  
W= 1470,960 kN  
Mass of footing  
m= 149,94 kN-sec<sup>2</sup>/m

Foundation dimensions  
Depth of foundation C= 1,2 m  
Length of foundation L= 10 m  
Width of foundation B= 5 m

Total weight (machinery+footing)  
W= 1667,09 kN

Rotation speed of machinery ω  
ω= 327 rpm 5,45 Hz

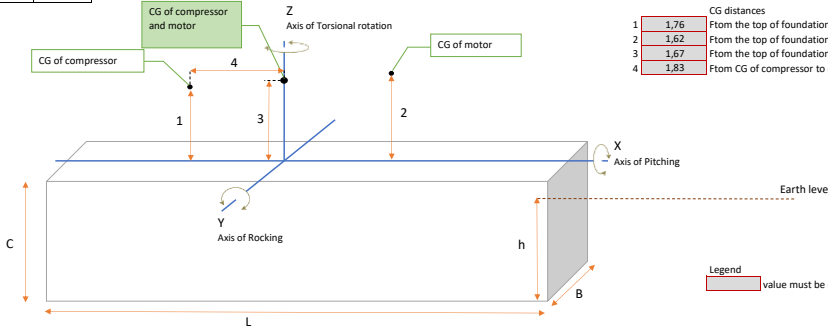
Soil parameters  
Soil density γ= 18 kN/m<sup>3</sup>  
Shear Modulus G= 21000,00 kN/m<sup>2</sup>  
Poisson's Ratio μ= 0,45  
Damping ξ= 0,05

No	Parameter	Formula	Vertical Z-direction	Horizontal X-direction	Rocking ψ-direction
1	Mass m, Mass moment of Inertia I	$m = \frac{W}{g}$ $g=9,81 \text{ m/sec}^2$ Mass moments of inertia are from Appendix 1	$m = 169,94$ kN-sec <sup>2</sup> /m	$m = 169,94$ kN-sec <sup>2</sup> /m	$I_{\psi}(\text{machine}) = 540,61$ $I_{\psi}(\text{footing}) = 330,38$ Total = 870,99 kN*m <sup>2</sup> -sec <sup>2</sup>
2	Equivalent radius r <sub>eq</sub> (m)	Table 1	3,99	3,99	3,39
3	Mass ratio B	Table 2	0,20	0,28	0,217
4	Spring constant coefficient α	Figure 11	L/B= 2,00 2,15	L/B= 2,00 0,95	L/B= 0,50 0,47
5	Spring constant k, (kN/m)	Table 3	5,80E+05	4,09E+05	4,49E+06
6	Damping ξ	Table 4	0,949	0,543	0,264
7	Total damping ξ	ξ <sub>s</sub> +ξ	0,95	0,54	0,31
8	Natural frequency f <sub>0</sub> (Hz)	$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$	9,31	7,81	11,43
9	Resonance frequency f <sub>res</sub> (Hz)	$f_{res} = \frac{f_0}{\sqrt{1-2\xi^2}}$	2ξ <sup>2</sup> > 1 resonance not possible	f <sub>res</sub> = 12,19	f <sub>res</sub> = 12,76
10	Compressor unbalanced inertia forces(kN) and moments(kN*m)	Machine parameter	V <sub>0</sub> = 2,793	H <sub>0</sub> = 62,488	T <sub>0</sub> = 324,938
11	Magnification factor	$M = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$ ; $r = \frac{\omega}{f_0}$	r <sub>1</sub> = 0,59 M <sub>1</sub> = 0,774	r <sub>1</sub> = 0,70 M <sub>1</sub> = 1,093	r <sub>1</sub> = 0,48 M <sub>1</sub> = 1,21
12	Vibration amplitude	$Z, X, \psi = \frac{\sum M(z, x, \psi) F(z, x, \psi)}{k(z, x, \psi)}$	0,000004 m or 0,003726 mm	0,0001670 m or 0,166965 mm	0,00008740 rad
13	Component of rocking oscillation, m		0,0002185012	0,0004544826	
14	Resultant vibration amplitude, mm	(12)+(13)	Z <sub>t</sub> = 0,2222268	X <sub>t</sub> = 0,6214472	
15	Transmissibility factor T	$T = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$	T <sub>1</sub> = 1,158	T <sub>1</sub> = 1,371	T <sub>1</sub> = 1,260
16	Transmitted force P	$P = \sum T \cdot F$	3,23 kN	85,68 kN	409,35 kN*m

Dynamic analyses

Actual soil pressure	Pstat= 33,34 kN/m <sup>2</sup>			
Must be < than 0,5 Static Allowable Bearing Capacity	Sall= 145 kN/m <sup>2</sup>	OK		
Dead load	1667,09 kN	P= 33,66 kN/m <sup>2</sup>	Must be < than 0,75 Static Allowable Bearing Capacity	OK
		P= 33,16 kN/m <sup>2</sup>	Must be < than 0,75 Static Allowable Bearing Capacity	OK
		P= 33,03 kN/m <sup>2</sup>	Must be > 0	NO UPLIFT
Resonant frequency	operating speed	5,45 less than	f < 0,8 f <sub>res</sub> or > 1,2 f <sub>res</sub>	OK
Rocking		10,21 or more than	15,31	
W <sub>footing</sub> =	7,50	Should be > or = 3,0 for well-balanced centrifugal machines.		OK
W <sub>machine</sub>				

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CG distances	
1	1,76 From the top of foundation to CG of compressor
2	1,62 From the top of foundation to CG of motor
3	1,67 From the top of foundation to combination CG of compressor and motor
4	1,83 From CG of compressor to combination CG of compressor and motor

Legend value must be entered manually

Density of concrete = 24,516 kN/m<sup>3</sup>

Static weight of machinery W= 82,87 kN  
 Mass of machinery m= 8,45 kN·sec<sup>2</sup>/m  
 Total weight (machinery+footing) W= 3515,11 kN

Soil parameters  
 Soil density γ= 17,59 kN/m<sup>3</sup>  
 Shear Modulus G= 103421,00 kN/m<sup>2</sup>  
 Poisson's Ratio μ= 0,35  
 Damping ζ= 0,05

Foundation dimensions  
 Depth of foundation C= 3,5 m  
 Length of foundation L= 8 m  
 Width of foundation B= 5 m  
 Depth of foundation embedment below grade h= 2 m

Rotation speed of machinery ω  
 Primary 1000 rpm 16,67 Hz  
 Secondary 900 rpm 15,00 Hz

No	Parameter	Formula	Vertical Z-direction	Horizontal X-direction	Rocking ψ-direction	Pitching θ-direction
1	Mass m, Mass moment of Inertia I	$m = \frac{W}{g}$ $I = 9,81 \text{ m/sec}^2$ Mass moments of inertia are from Appendix 1	m = 358,32 kN·sec <sup>2</sup> /m	m = 358,32 kN·sec <sup>2</sup> /m	I <sub>ψ(machinery)</sub> = 225,79 I <sub>ψ(footing)</sub> = 2223,14 Total = 2448,93 kN·m <sup>2</sup> /sec <sup>2</sup>	I <sub>θ(machinery)</sub> = 225,79 I <sub>θ(footing)</sub> = 1086,06 Total = 1311,85 kN·m <sup>2</sup> /sec <sup>2</sup>
Spring constant calculation						
2	Equivalent radius r <sub>eq</sub> (m)	Table 1	3,57	3,57	4,06	3,21
3	Embedment coefficients for spring constants η	Table 5	1,22	1,51	1,42	1,55
4	Spring constant coefficient α	Figure 11	L/B= 1,60 2,15	L/B= 1,60 0,95	L/B= 1,60 0,58	L/B= 0,63 0,45
5	Spring constant k (kN/m)	Table 6	2,64E+06	2,53E+06	4,21E+07	2,22E+07
Damping calculation						
6	Damping ratio embedment factor α	Table 7	1,533	2,244	1,125	1,223
7	Mass ratio B	Table 2	0,71	0,89	0,302	0,52
8	Effective damping coefficient	Figure 12	-	-	1,600	1,630
9	Damping ζ	Table 8	0,771	0,686	0,16	0,11
10	Total damping ζ	ζ*α	0,82	0,74	0,21	0,16
11	Natural frequency f <sub>0</sub> (Hz)	$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$	13,66	13,38	20,87	20,71
12	Resonance frequency f <sub>res</sub> (Hz)	$f_{res} = \frac{f_0}{\sqrt{1-2\zeta^2}}$	resonance not possible	resonance not possible	f <sub>res</sub> = 21,89	f <sub>res</sub> = 21,24
13	Compressor unbalanced inertia forces(kN) and moments(kN*m)	Machine parameter	F <sub>1</sub> = 0,000 F <sub>2</sub> = 0,000	F <sub>1</sub> = 1,970 F <sub>2</sub> = 0,947	M <sub>1</sub> = 20,280 M <sub>2</sub> = 0,1166	M <sub>1</sub> = 24,400 M <sub>2</sub> = 0,000
14	Magnification factor	$M = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$ ; $r = \frac{\omega}{f_0}$	r <sub>1</sub> = 1,22 r <sub>2</sub> = 1,10 M <sub>1</sub> = 0,48 M <sub>2</sub> = 0,55	r <sub>1</sub> = 1,25 r <sub>2</sub> = 0,52 M <sub>1</sub> = 0,80 M <sub>2</sub> = 0,72	r <sub>1</sub> = 0,80 r <sub>2</sub> = 0,72 M <sub>1</sub> = 2,01 M <sub>2</sub> = 1,75	r <sub>1</sub> = 0,80 r <sub>2</sub> = 0,72 M <sub>1</sub> = 2,30 M <sub>2</sub> = 1,90
15	Displacement Response	$Z, X, \psi, \theta = \frac{\sum M(x, x, \psi, \theta) F(x, x, \psi, \theta)}{k(x, x, \psi, \theta)}$	0,0000000 m or 0,0000000 mm	0,00000004 m or 0,000407 mm	0,0000010 rad	0,0000025 rad
16	Total displacement, mm	Zt = Z + ψ · R <sub>h</sub> x + θ · R <sub>h</sub> y Xt = X + ψ · R <sub>v</sub> Yt = θ · R <sub>v</sub>	Z = 0,010228	X = 0,005439853 Y = 0,01309733		
17	Transmissibility factor T	$T = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$	T <sub>1</sub> = 1,086 T <sub>2</sub> = 1,136	T <sub>1</sub> = 1,091	T <sub>1</sub> = 2,123 T <sub>2</sub> = 1,827	T <sub>1</sub> = 2,377 T <sub>2</sub> = 1,945
18	Transmitted force P	$P = \sum T \cdot F$	0,00 kN	2,15 kN	43,27 kN*m	58,00 kN*m

Dynamic analyses

Transmitted Dynamic vertical force	P <sub>v</sub> =	0,00	kN
Transmitted Moments	P <sub>r</sub> =	54,57	kN*m
Dead load	P <sub>dyn</sub> =	0,07	kN/m <sup>2</sup>
		-0,07	kN/m <sup>2</sup>
	P <sub>stat</sub> =	87,88	kN/m <sup>2</sup>
Total Downward pressure		87,95	kN/m <sup>2</sup>
Must be < than 0,75 Static Allowable Bearing Capacity		145	OK
Total Upward pressure		87,81	kN/m <sup>2</sup>
Must be > 0		NO UPLIFT	
Resonant frequency			
Rocking	operating speed	16,67 less than	17,51 or more than
Pitching	operating speed	15,00 less than	17,00 or more than